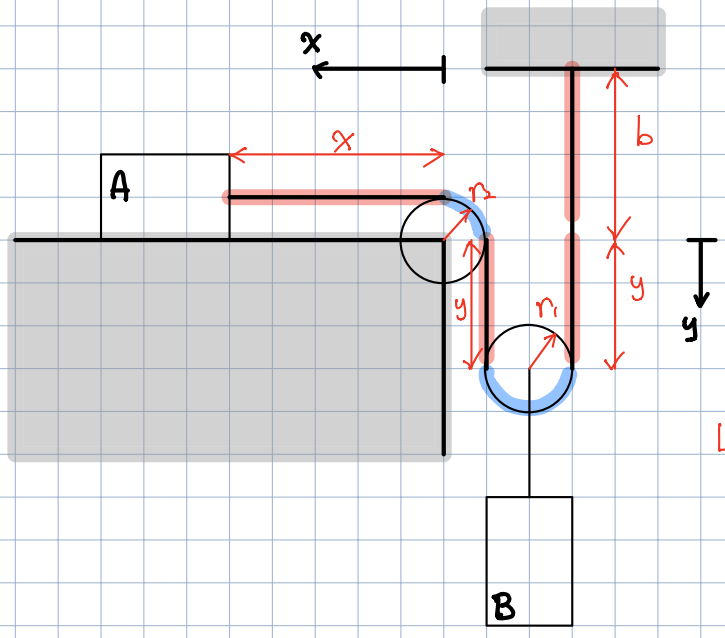
Constrained Motion of Connected Particles

1. One Degree of Freedom



The length of the cable :

$$L = x + \frac{2\pi r_2}{4} + 2y + \frac{2\pi r_1}{2} + b$$

$$= x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

L, r_2, r_1, b = constant

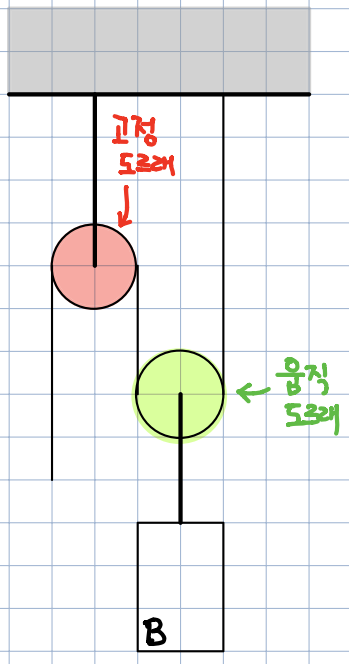
미분

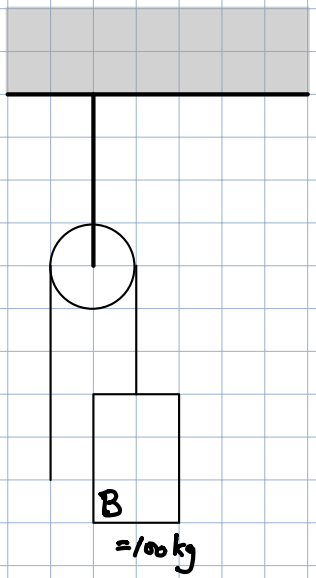
$$0 = \dot{x} + 2\dot{y}$$

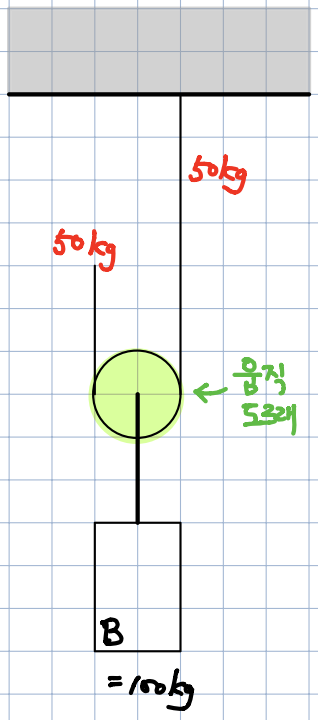
$$0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y}$$

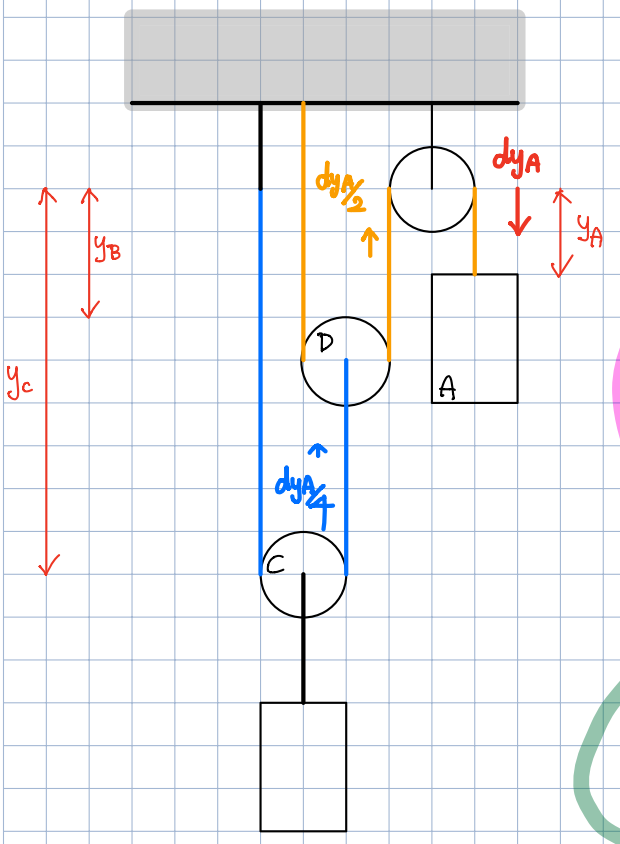
$$0 = a_A + 2a_B$$







2. Two Degree of Freedom



L_A : $L_A = y_A + 2y_D + \text{constant}$

$0 = \dot{y}_A + 2\dot{y}_D$

$0 = \ddot{y}_A + 2\ddot{y}_D$

L_B : $L_B = y_B + y_C + (y_C - y_D) + \text{constant}$

$0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$

$0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$

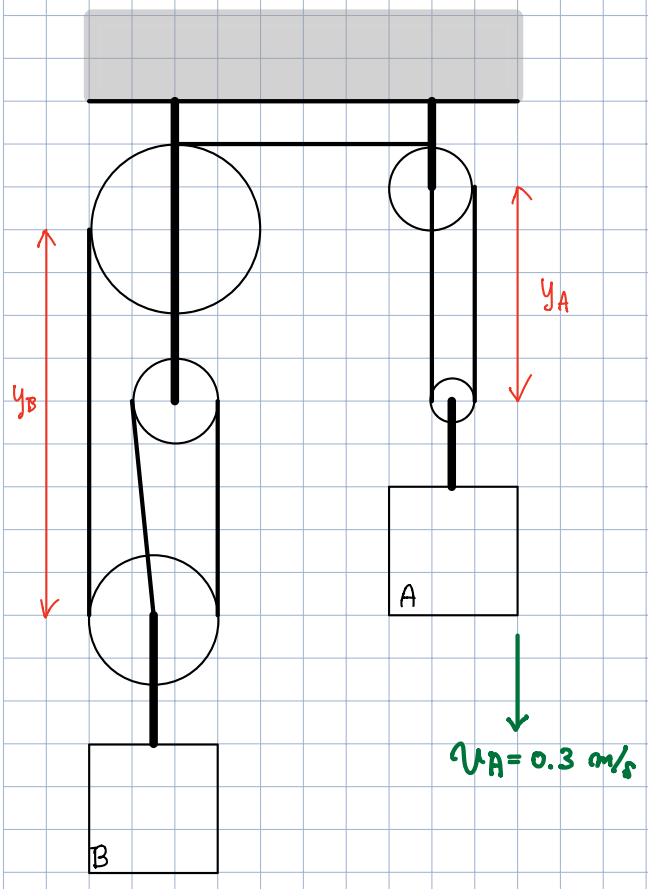
$2\dot{y}_D = 2\dot{y}_B + 4\dot{y}_C$

$0 = \dot{y}_A + 2\dot{y}_B + 4\dot{y}_C \longrightarrow v_A + 2v_B + 4v_C = 0$

$\ddot{y}_D = \ddot{y}_B + 2\ddot{y}_C$

$0 = \ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C \longrightarrow a_A + 2a_B + 4a_C = 0$

Example > 2.15



A has a downward velocity of 0.3 m/s

Determine the velocity of B.

$L = 2y_A + 3y_B + \text{const}$

$0 = 2\dot{y}_A + 3\dot{y}_B \longleftarrow \dot{y}_A = v_A = 0.3 \ (\downarrow)$

$0 = 2v_A + 3v_B$

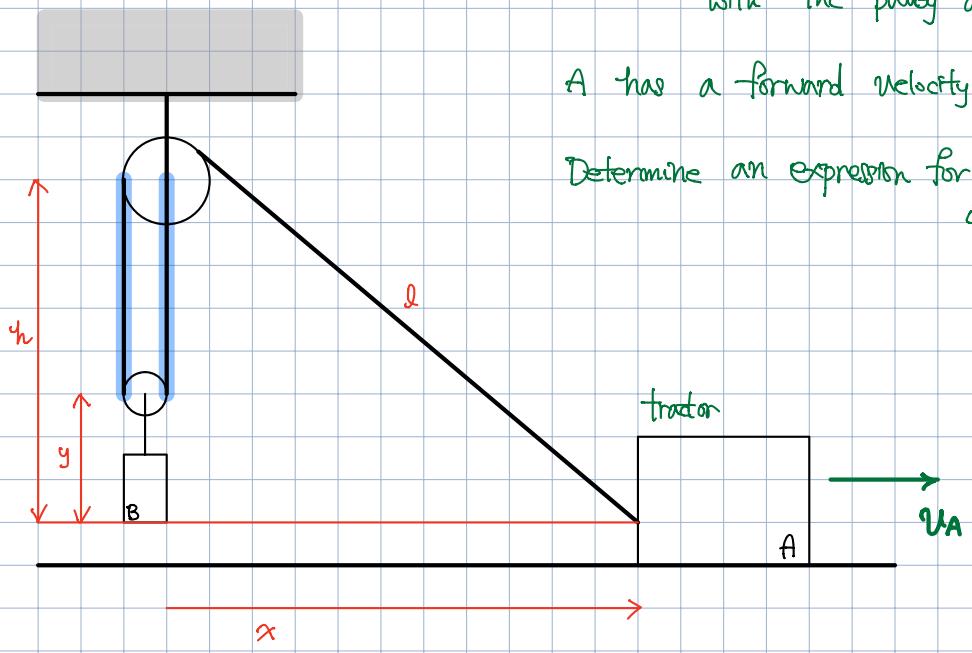
$v_B = -\frac{2}{3} \cdot v_A = -0.2 \text{ m/s}$ Ans.

Example > 2.16

The tractor A is used to hoist B with the pulley arrangement

A has a forward velocity v_A

Determine an expression for upward velocity v_B in terms of x



$$L = l + 2\cdot(h-y)$$

$$0 = \dot{l} - 2\dot{y} \quad \longleftarrow \quad l = \sqrt{x^2+h^2}$$

$$\dot{l} = \frac{dl}{dt} = \frac{dl}{dx}\cdot\frac{dx}{dt}$$

$$0 = \frac{x\cdot\dot{x}}{\sqrt{x^2+h^2}} - 2\dot{y}$$

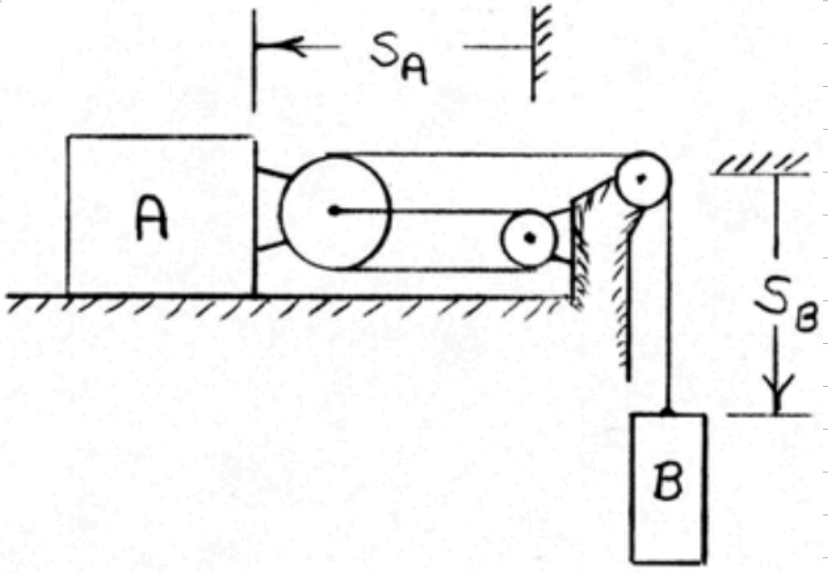
$$dl/dx = \frac{2x}{2\sqrt{x^2+h^2}}$$

$$v_B = \dot{y} = \frac{x\cdot v_A}{\sqrt{x^2+h^2}}$$

Ans.

$$\dot{l} = \frac{x}{\sqrt{x^2+h^2}}\cdot\frac{dx}{dt}$$

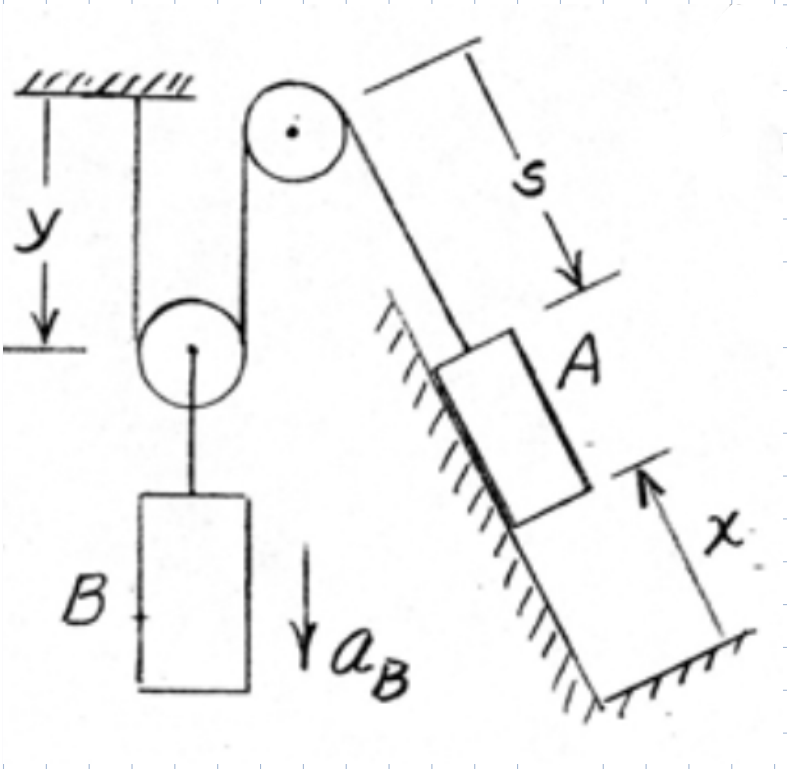
$$= \frac{x}{\sqrt{x^2+h^2}}\cdot\dot{x}$$



$L = S_B + 3S_A + \text{const}$

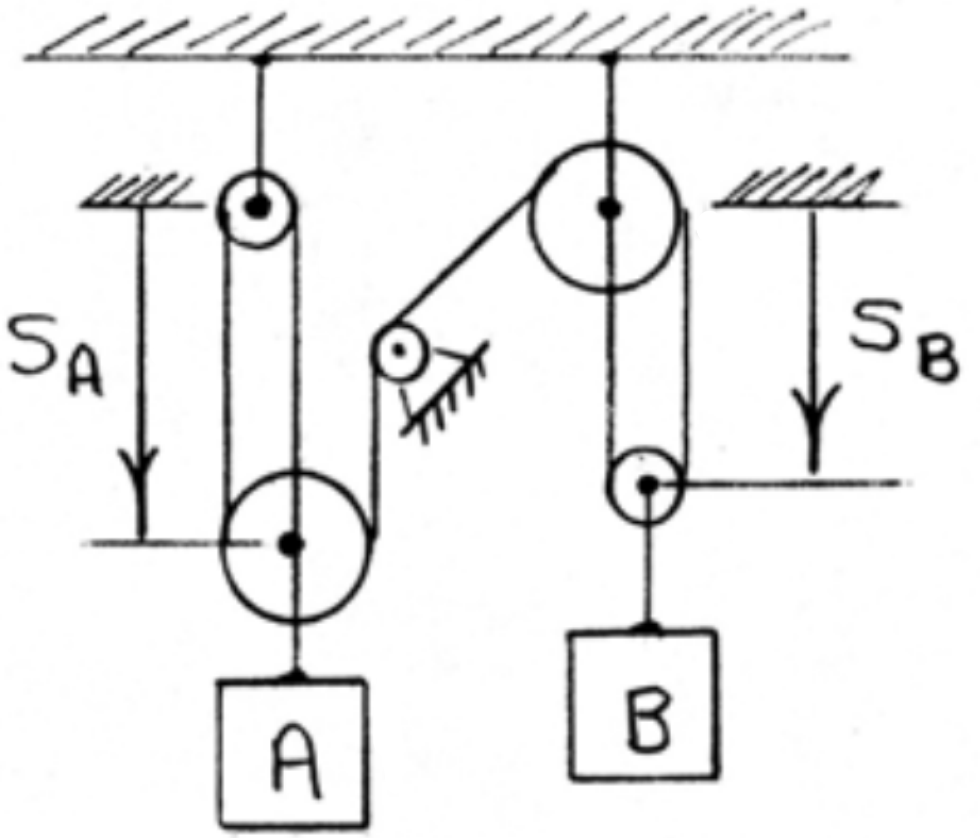
$0 = v_B + 3v_A$

$\therefore v_B = -3v_A$ Ans.



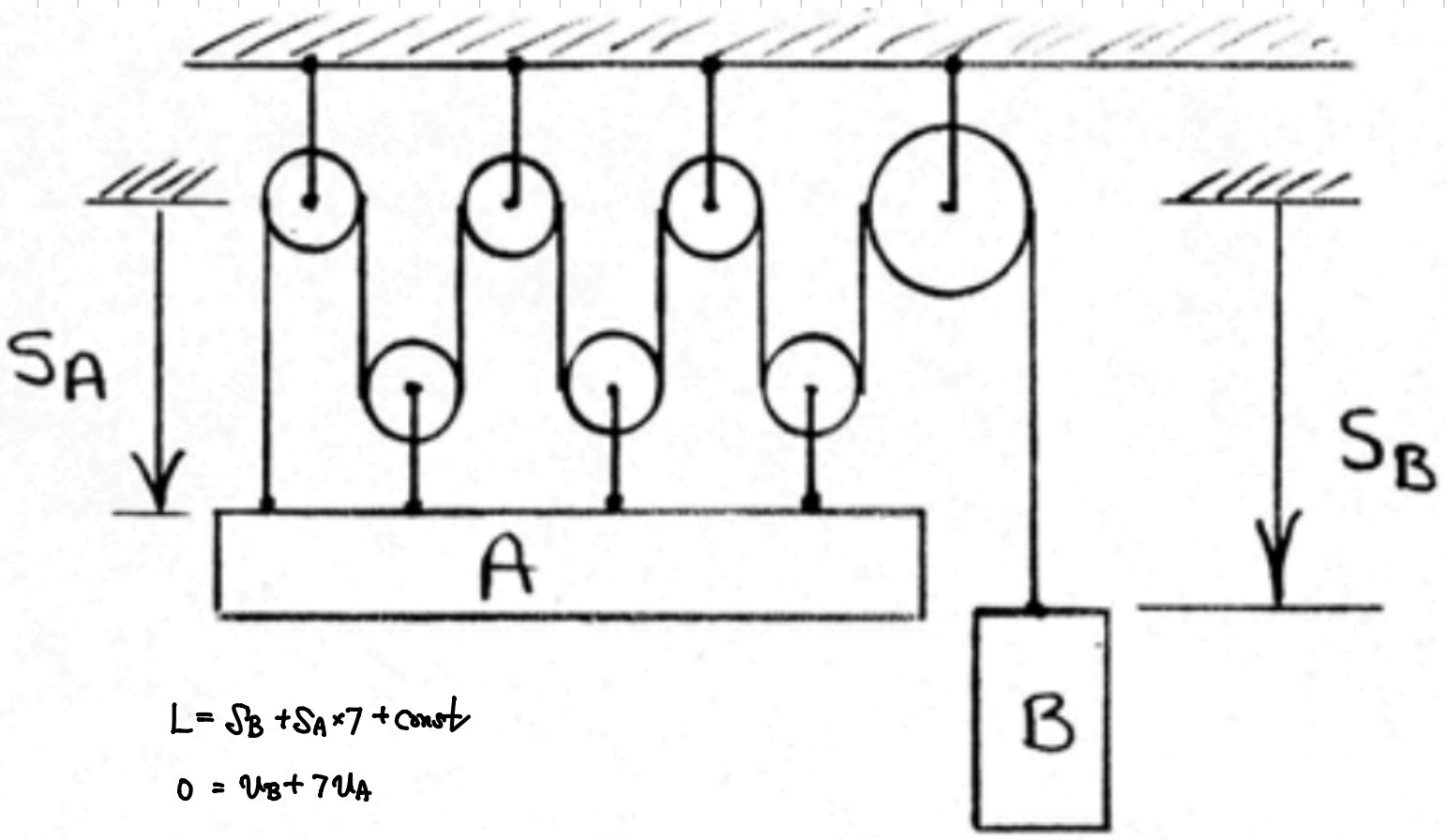
$L = 2y + s + \text{constant}$

$0 = 2\dot{y} + \dot{s}$ Ans.



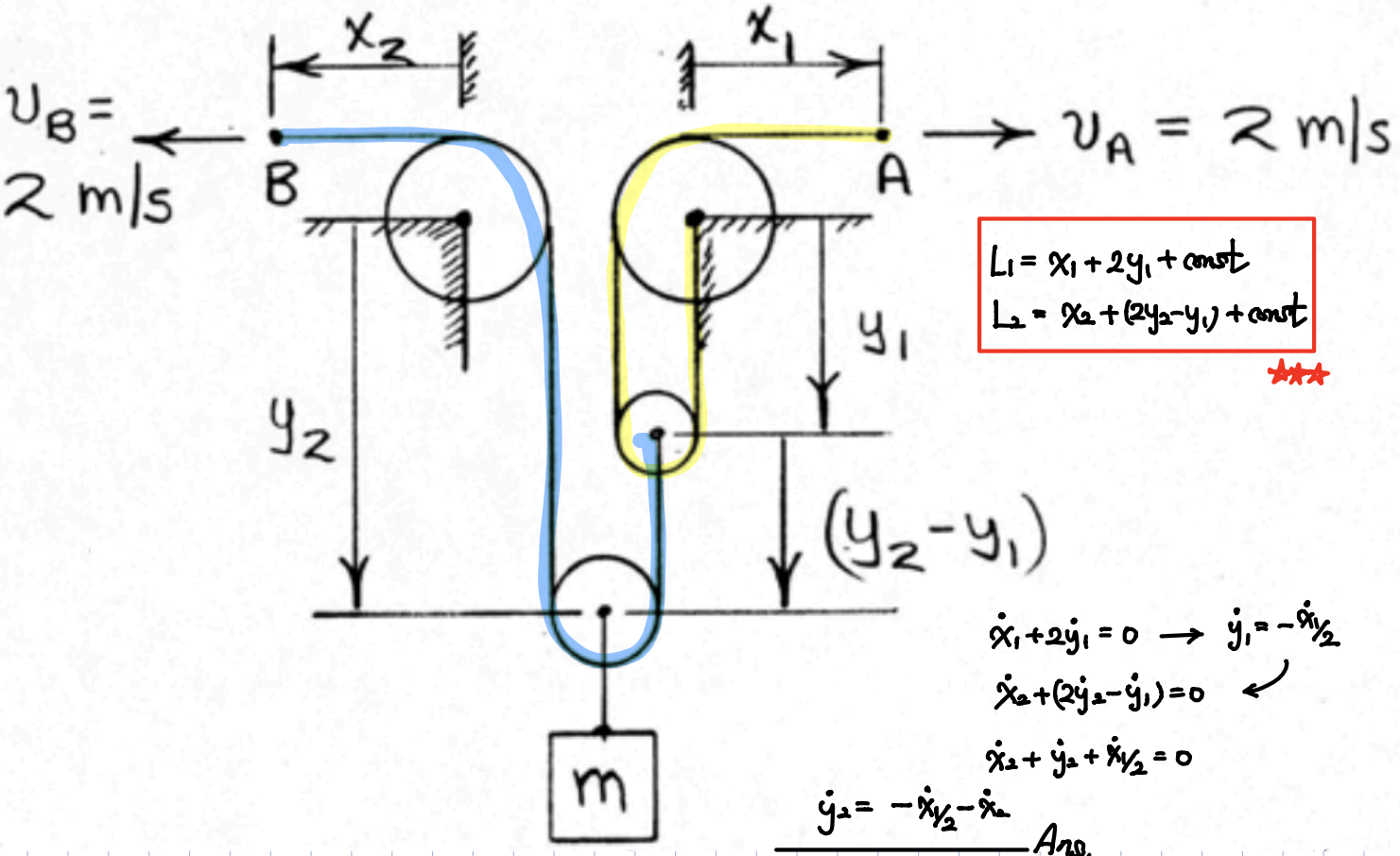
$L = 3S_A + 2S_B + \text{const}$

$0 = 3v_A + 2v_B$ Ans.



$L = S_B + S_A \times 7 + \text{const}$

$0 = v_B + 7v_A$



$L_1 = x_1 + 2y_1 + \text{const}$

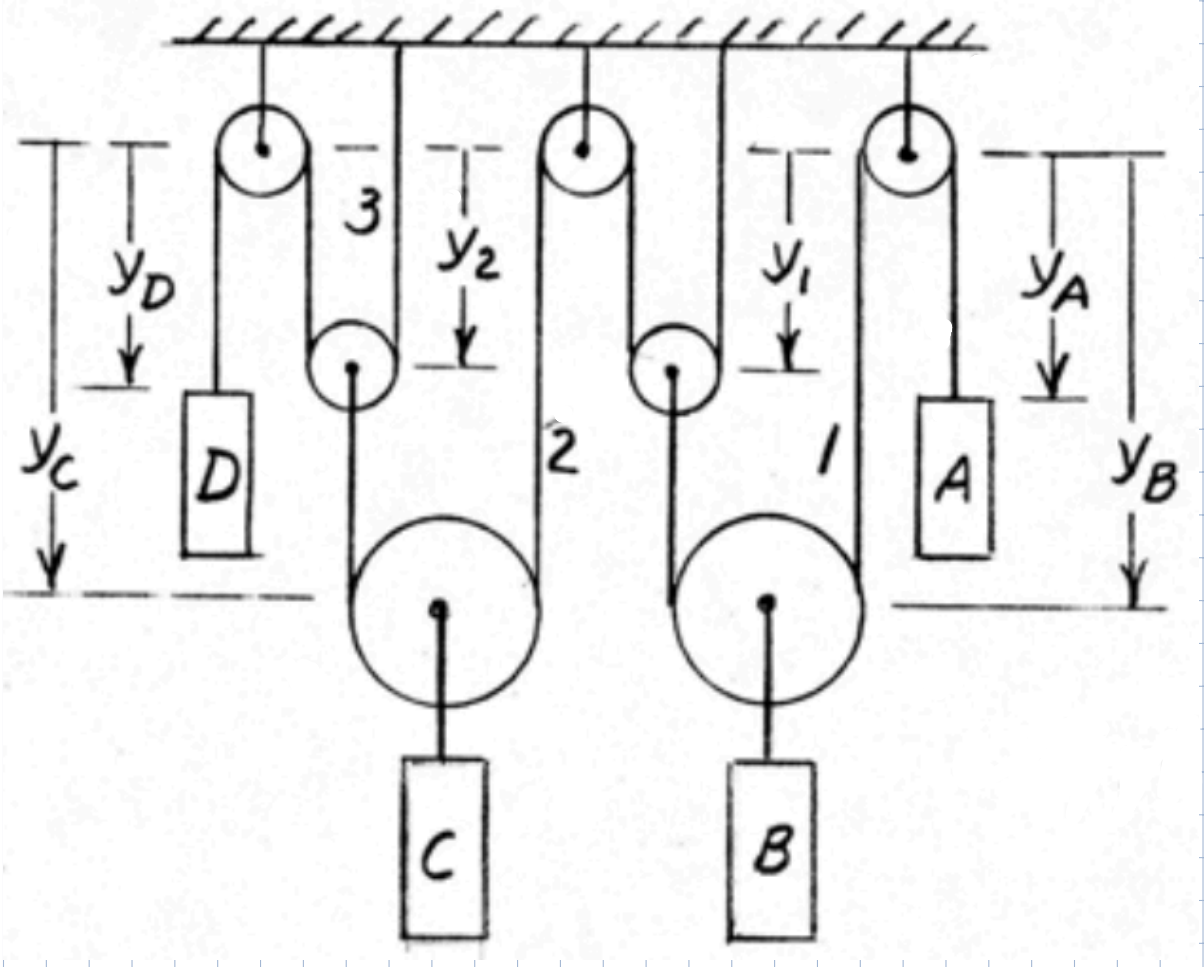
$L_2 = x_2 + (2y_2 - y_1) + \text{const}$ ***

$\dot{x}_1 + 2\dot{y}_1 = 0 \rightarrow \dot{y}_1 = -\dot{x}_1/2$

$\dot{x}_2 + (2\dot{y}_2 - \dot{y}_1) = 0$

$\dot{x}_2 + \dot{y}_2 + \dot{x}_1/2 = 0$

$\dot{y}_2 = -\dot{x}_1/2 - \dot{x}_2$ Ans.


3
2
1
y_D
y_2
y_1
y_A
y_C
y_B
D
C
B
A