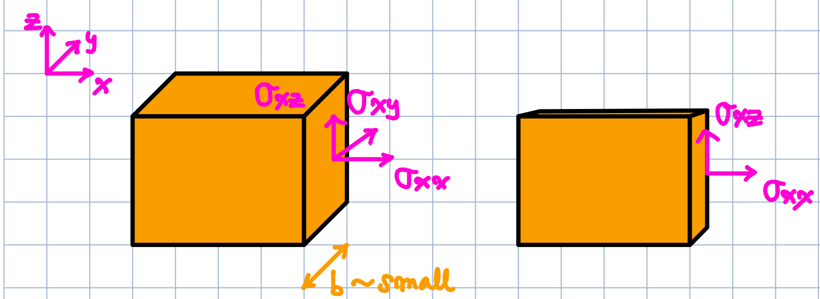
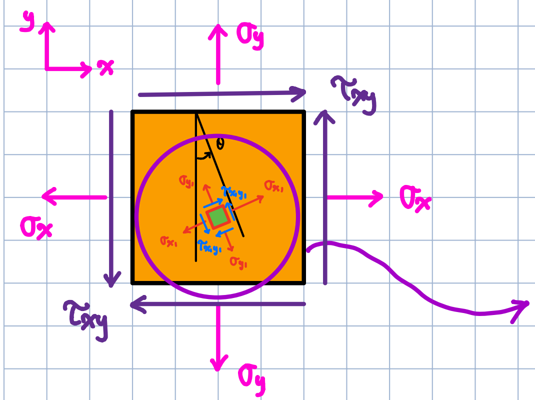
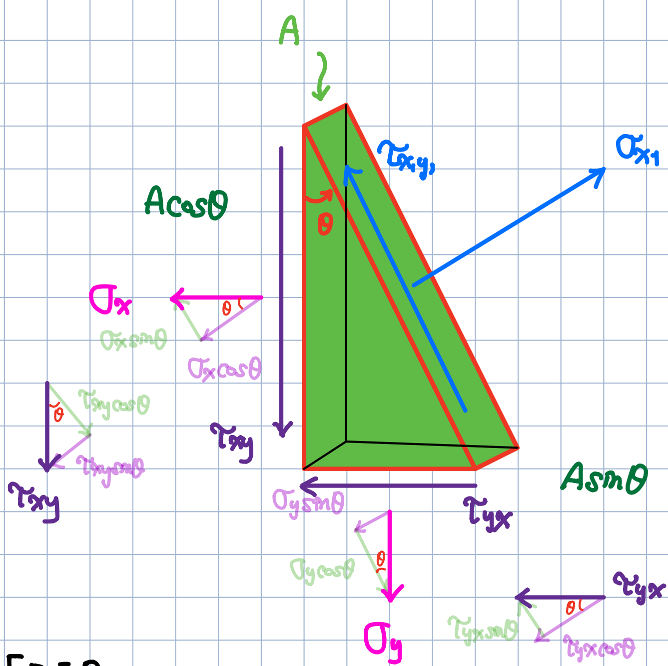
20200910

Mohr's Circle



1. Equilibrium





Equilibrium : $\Sigma F_{\swarrow} = 0$, $\Sigma F_{\nwarrow} = 0$

i) $\Sigma F_{x_1} = \sigma_{x_1} \cdot \cancel{A} - \sigma_x \cos\theta \cdot \cancel{A} \cos\theta - \tau_{xy} \sin\theta \cdot \cancel{A} \cos\theta$

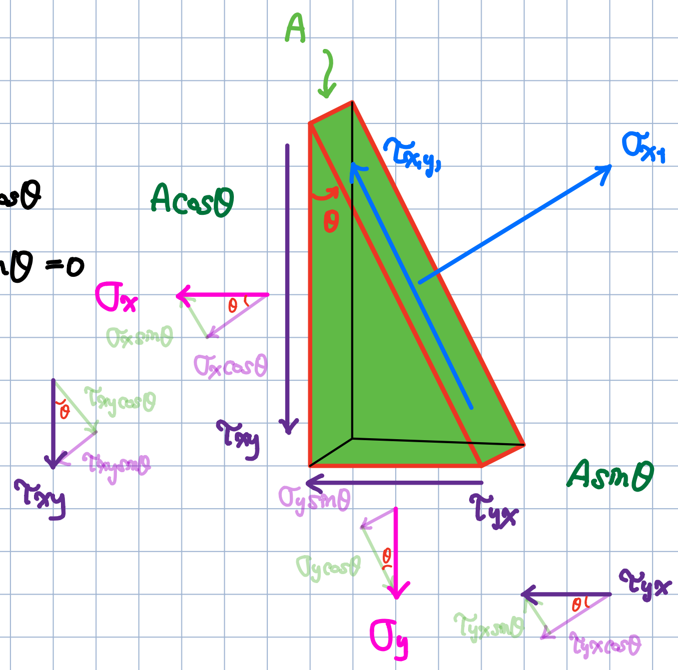
$- \sigma_y \sin\theta \cdot \cancel{A} \sin\theta - \tau_{yx} \cos\theta \cdot \cancel{A} \sin\theta = 0 \quad \leftarrow \quad \tau_{xy} = \tau_{yx}$

$$\sigma_{x_1} = \sigma_x \underbrace{\cos^2\theta}_{= \frac{1+\cos 2\theta}{2}} + \sigma_y \underbrace{\sin^2\theta}_{= \frac{1-\cos 2\theta}{2}} + \underbrace{2 \tau_{xy} \sin\theta\cos\theta}_{= \sin 2\theta}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Ans.

iii) $\Sigma F_{x_1y_1} = 0$



$$\Sigma F_{x_1y_1} = \tau_{x_1y_1}\cdot\cancel{A} + \sigma_x\sin\theta\cdot\cancel{A}\cos\theta - \tau_{xy}\cos\theta\cdot\cancel{A}\cos\theta - \sigma_y\cos\theta\cdot\cancel{A}\sin\theta + \tau_{xy}\sin\theta\cdot\cancel{A}\sin\theta = 0$$

$$\tau_{x_1y_1} = -\sigma_x\underbrace{\sin\theta\cos\theta}_{=\frac{\sin2\theta}{2}} + \sigma_y\underbrace{\cos\theta\sin\theta}_{=\frac{\sin2\theta}{2}} + \tau_{xy}(\underbrace{\cos^2\theta-\sin^2\theta}_{=\cos2\theta})$$

$$\tau_{x_1y_1} = -\frac{\sigma_x-\sigma_y}{2}\sin2\theta + \tau_{xy}\cos2\theta$$

Ans.

$$\sigma_{x_1} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos2\theta + \tau_{xy}\sin2\theta$$

$$\tau_{x_1y_1} = -\frac{\sigma_x-\sigma_y}{2}\sin2\theta + \tau_{xy}\cos2\theta$$

$$\sigma_{x_1} - \left(\frac{\sigma_x+\sigma_y}{2}\right) = \frac{\sigma_x-\sigma_y}{2}\cos2\theta + \tau_{xy}\sin2\theta$$

$(a+b)^2 = a^2+2ab+b^2$

$$\tau_{x_1y_1} = -\frac{\sigma_x-\sigma_y}{2}\sin2\theta + \tau_{xy}\cos2\theta$$

$$\left\{\sigma_{x_1} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right\}^2 = \left\{\frac{\sigma_x-\sigma_y}{2}\cos2\theta + \tau_{xy}\sin2\theta\right\}^2$$

$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + \tau_{xy}^2\sin^2 2\theta + 2\cdot\frac{\sigma_x-\sigma_y}{2}\cos2\theta\cdot\tau_{xy}\sin2\theta$$

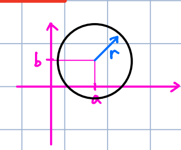
$$\tau_{x_1y_1}^2 = \left\{-\frac{\sigma_x-\sigma_y}{2}\sin2\theta + \tau_{xy}\cos2\theta\right\}^2$$

$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\sin^2 2\theta + \tau_{xy}^2\cos^2 2\theta - 2\,\frac{\sigma_x-\sigma_y}{2}\sin2\theta\cdot\tau_{xy}\cos2\theta$$

$$\left\{\sigma_{x_1} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right\}^2 + \tau_{x_1y_1}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2(\underbrace{\cos^2 2\theta+\sin^2 2\theta}_{=1}) + \tau_{xy}^2(\underbrace{\sin^2 2\theta+\cos^2 2\theta}_{=1})$$

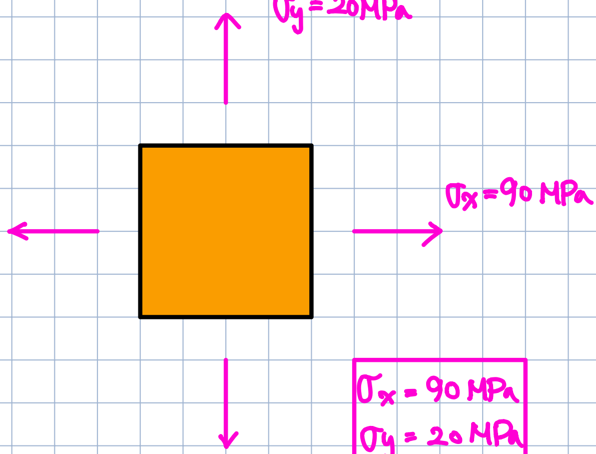
$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$(x-a)^2+(y-b)^2 = r^2$



Example 1.

Using Mohr's circle → determine the stresses acting on a plane at $\theta = 30^\circ$



$$\sigma_x = 90\,MPa$$
$$\sigma_y = 20\,MPa$$
$$\tau_{xy} = 0\,MPa$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Mohr's circle :

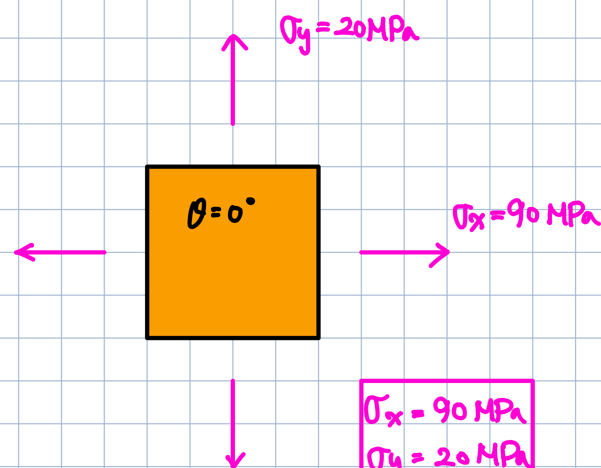
$$\left\{\sigma_{x_1} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right\}^2 + \tau_{x_1y_1}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$C: \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$C = \frac{90 + 20}{2} = 110/2 = 55\,MPa$$

$$R = \sqrt{\left(\frac{90 - 20}{2}\right)^2 + 0^2} = 70/2 = 35\,MPa$$

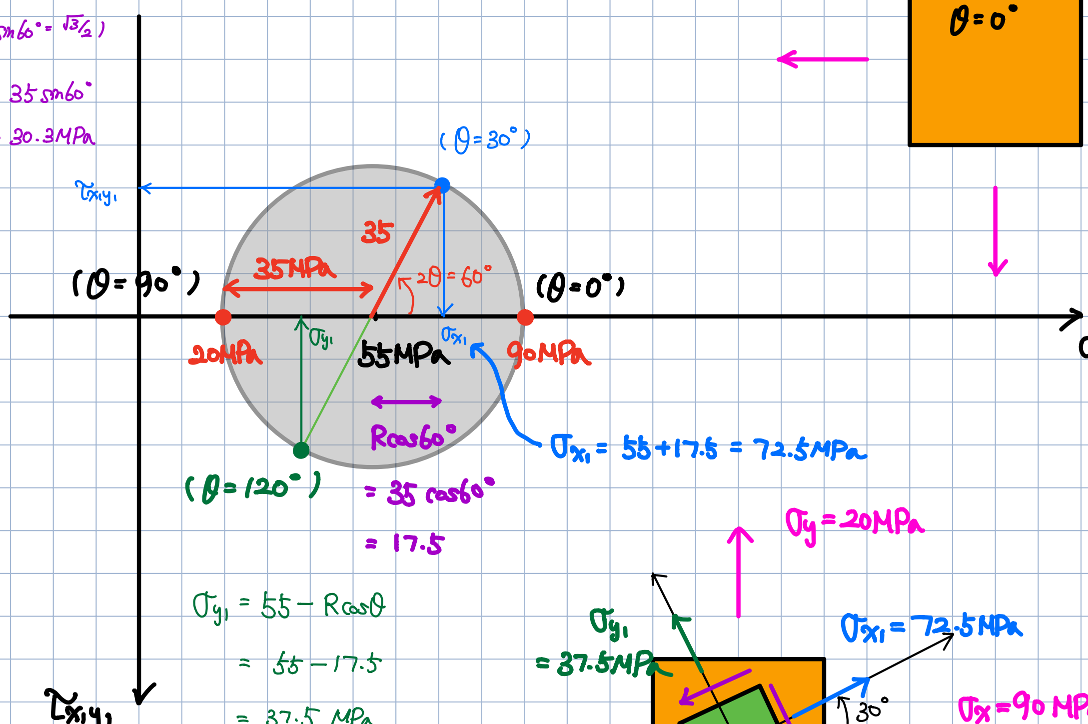


$$\sigma_x = 90\,MPa$$
$$\sigma_y = 20\,MPa$$
$$\tau_{xy} = 0\,MPa$$

$(\sin 60^\circ = \sqrt{3}/2)$

$\tau_{x_1y_1} = -35\sin 60^\circ$

$= -30.3\,MPa$



$\sigma_{x_1} = 55 + 17.5 = 72.5\,MPa$

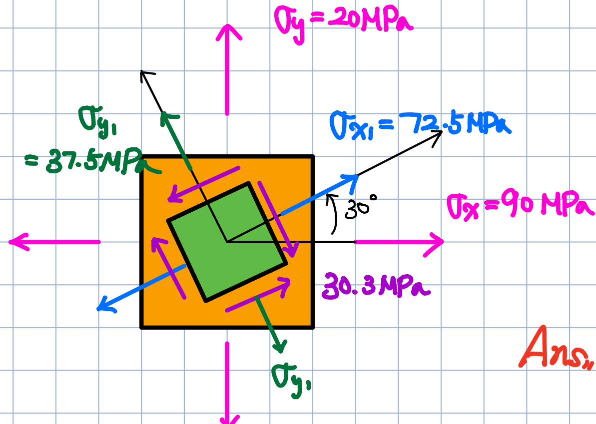
$= 35\cos 60^\circ$

$= 17.5$

$\sigma_{y_1} = 55 - R\cos\theta$

$= 55 - 17.5$

$= 37.5\,MPa$



Ans.