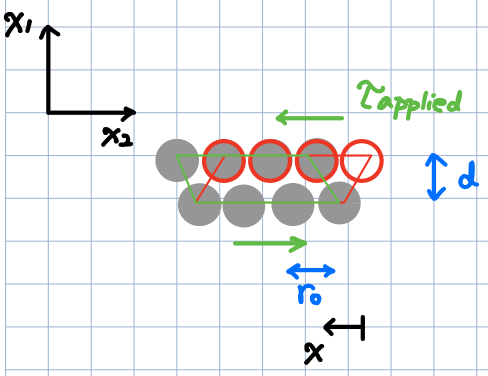
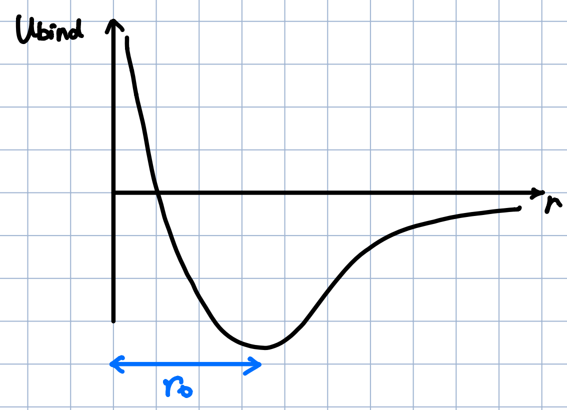
Ideal or theoretical shear strength





constant

$$\tau \sim k \cdot \frac{\sin 2\pi x}{r_0}$$

for small x , $\sin 2\pi x / r_0 \sim 2\pi x / r_0$

$$\tau = \frac{k 2\pi x}{r_0}$$

$$\gamma = 2\varepsilon_{12} \sim x/d \longrightarrow \tau = G\gamma \sim Gx/d$$

$$\rightarrow Gx/d \sim \frac{k 2\pi x}{r_0} \longrightarrow k \sim G r_0 / 2\pi d$$

$$\therefore \tau = G r_0 / 2\pi d \cdot \frac{\sin 2\pi x}{r_0}$$

Ans.

$\tau_{max} = G/2\pi$ when $\frac{\sin 2\pi x}{r_0} = 1$

$d \sim r_0$ (order)

$\gamma_{Th} = 2\tau_{Th} \sim G/5$: Theoretical strengths

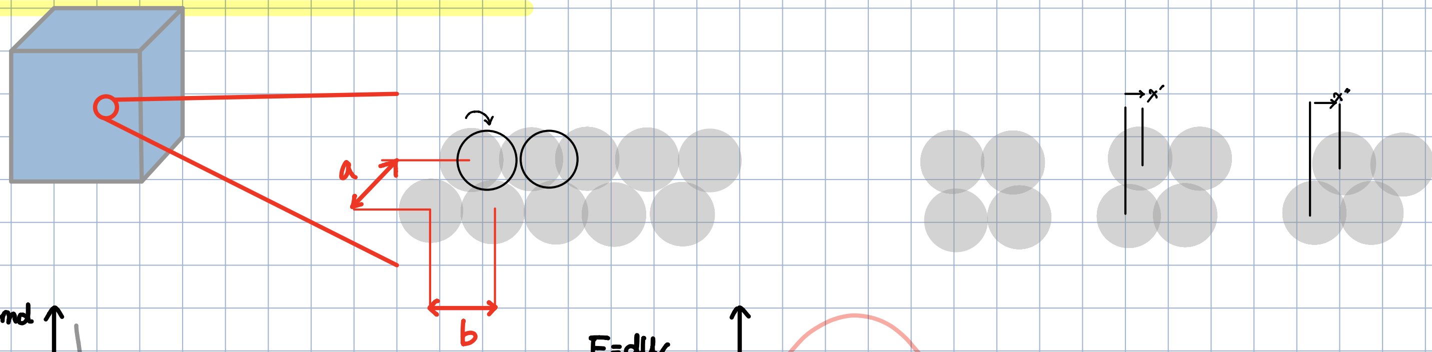
2 orders of magnitude

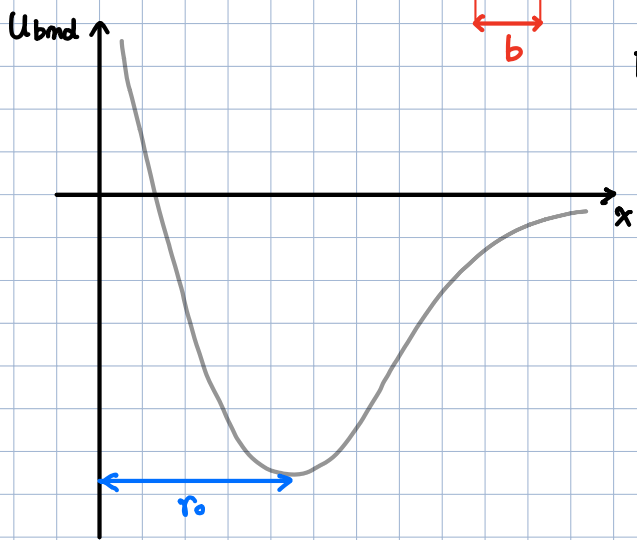
$\longrightarrow$ defects in materials

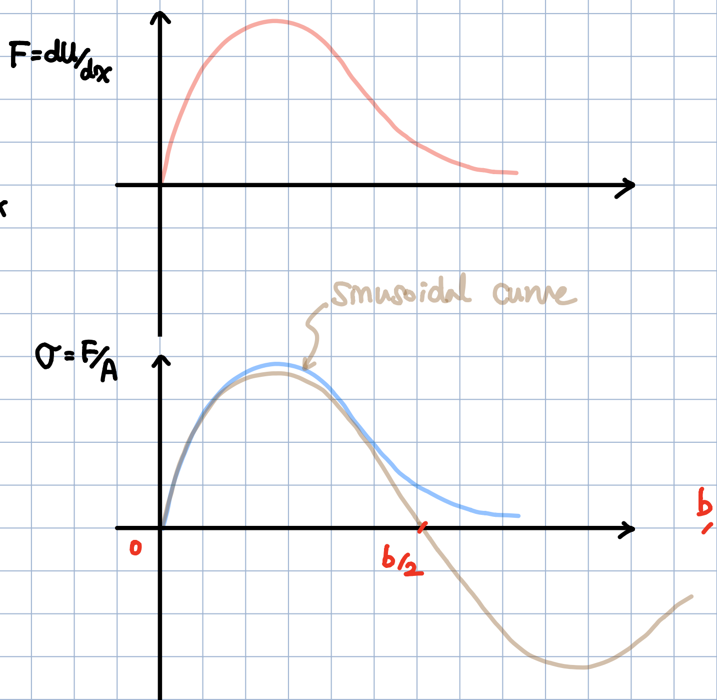
$\tau_{Th} \gg \tau_{exp}$ $\because$ dislocations ★★

$\tau_{Th}^{cohesive} > \tau_F$

Theoretical / ideal of shear stress







U = energy, Work

$W = F \cdot x$

$dU/dx = F$

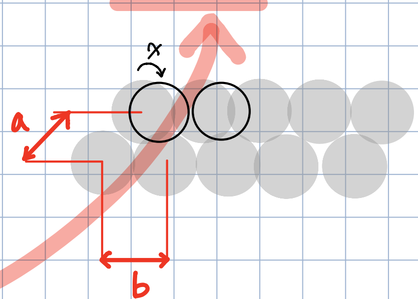
$$\tau = \tau_{max} \sin\left(\frac{2\pi x}{b}\right)$$

At small values of strain $\rightarrow$ $\tau = G\gamma$ $\rightarrow$ $d\tau/d\gamma = G$

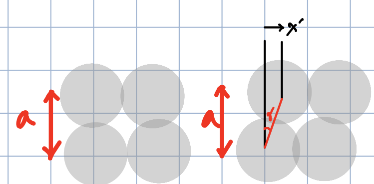
- $d\tau/dx = \tau_{max} \cdot 2\pi/b \cdot \cos\left(\frac{2\pi x}{b}\right)$ $\longrightarrow$ $(d\tau/dx)_{x\approx 0} = \dfrac{2\pi \tau_{max}}{b}$
- $d\tau/dx = d\tau/d\gamma \cdot d\gamma/dx = G \cdot \frac{d}{dx}(\gamma)$

$= G \cdot \frac{d}{dx}(x/a)$

$= G \cdot 1/a$



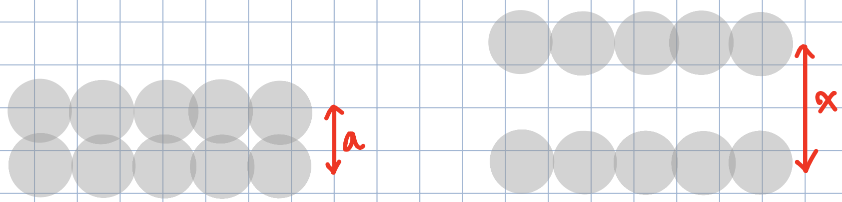
$\tan\gamma \approx \gamma = x/a$

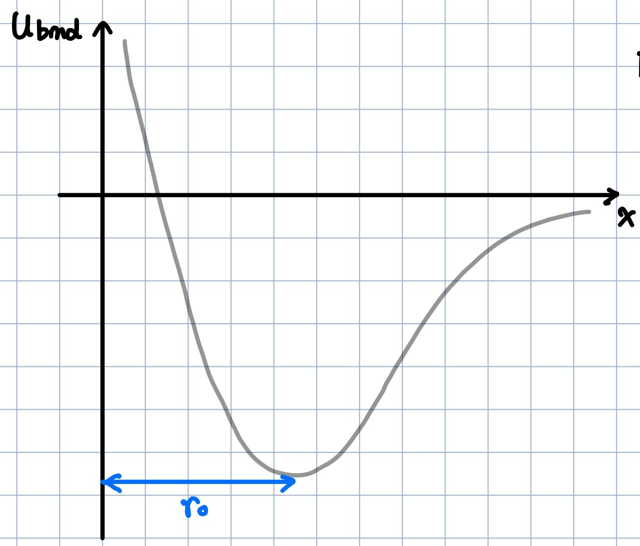


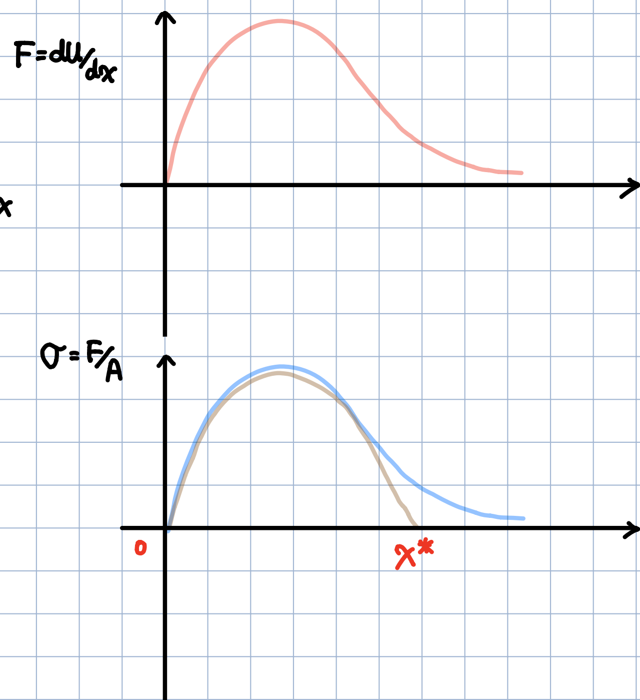
$$\therefore \frac{2\pi\tau_{max}}{b} = G \cdot 1/a$$

$\therefore \tau_{max} = \dfrac{Gb}{2\pi a}$ if $a = b$ $\longrightarrow$ $\tau_{max} = \dfrac{G}{2\pi}$ Ans.

Theoretical Normal Stress







U = energy, Work

$W = F \cdot x$

$dU/dx = F$

$$\sigma = \sigma_t \cdot \sin(\pi x / x^*)$$

$$\frac{d\sigma}{dx} = \sigma_t \cdot \frac{\pi}{x^*} \cdot \cos(\pi x / x^*) \longleftarrow x \approx 0$$

$$= \sigma_t \cdot (\pi / x^*)$$

$$\sigma = E\varepsilon \longrightarrow d\sigma/d\varepsilon = E$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{x}{a} \longrightarrow d\varepsilon/dx = 1/a$$

$$\therefore \frac{d\sigma}{dx} = \frac{d\sigma}{d\varepsilon} \cdot \frac{d\varepsilon}{dx} = E \cdot 1/a = E/a$$

$$\therefore \sigma_t \cdot \pi/x^* = E/a$$

$$\therefore \sigma_t = E/\pi \cdot x^*/a$$

at $x^* \approx a \longrightarrow \sigma_t = E/\pi$ Ans.