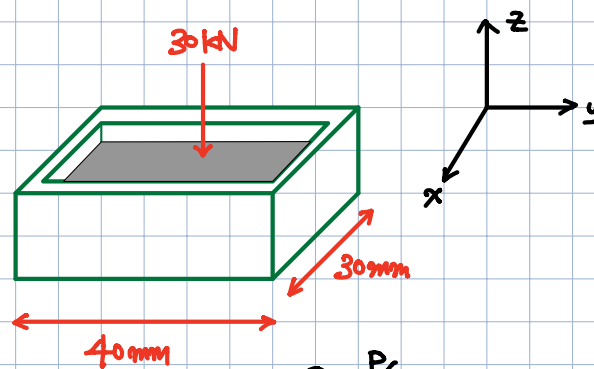


Predicting yield using Tresca criteria

UHMWPE block

load = 30 kN, $E = 1 \text{ GPa}$, $\nu = 0.4$



a) What are the stresses and strains?

$$\epsilon_x = \frac{1}{E} \{ \sigma_x - \nu(\sigma_y + \sigma_z) \} \quad \leftarrow \epsilon_x = 0$$

$$\epsilon_y = \frac{1}{E} \{ \sigma_y - \nu(\sigma_z + \sigma_x) \} \quad \leftarrow \epsilon_y = 0$$

$$\epsilon_z = \frac{1}{E} \{ \sigma_z - \nu(\sigma_x + \sigma_y) \}$$

$$\sigma_z = \frac{P}{A} = \frac{-30,000 \text{ N}}{(0.03 \text{ m}) \times (0.04 \text{ m})} = -25 \text{ MPa}$$

$$\begin{aligned} \sigma_x - \nu(\sigma_y + \sigma_z) &= 0 & \leftarrow \sigma_y = ? \\ \nu\sigma_y - \nu^2(\sigma_z + \sigma_x) &= 0 \end{aligned}$$

$$\sigma_x - \nu\sigma_z - \nu^2\sigma_z - \nu^2\sigma_x = 0$$

$$\sigma_x(1 - \nu^2) = \nu(1 + \nu)\sigma_z$$

$$\begin{aligned} \therefore \sigma_x &= \frac{\nu(1 + \nu)\sigma_z}{1 - \nu^2} = \frac{\nu(1 + \nu)\sigma_z}{(1 - \nu)(1 + \nu)} \\ &= \frac{\nu}{1 - \nu} \cdot \sigma_z \quad \leftarrow \nu = 0.4, \sigma_z = -25 \text{ MPa} \\ &= \frac{0.4}{1 - 0.4} \cdot (-25 \text{ MPa}) \\ &= -16.7 \text{ MPa} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} \nu\sigma_x - \nu^2(\sigma_y + \sigma_z) &= 0 \\ \sigma_y - \nu(\sigma_z + \sigma_x) &= 0 \end{aligned}$$

$$-\nu^2\sigma_y - \nu^2\sigma_z + \sigma_y - \nu\sigma_z = 0$$

$$\sigma_y(1 - \nu^2) = \nu(1 + \nu)\sigma_z$$

$$\begin{aligned} \therefore \sigma_y &= \frac{\nu(1 + \nu)}{1 - \nu^2} \sigma_z \\ &= \frac{\nu}{1 - \nu} \sigma_z \quad \text{Ans.} \\ &= -16.7 \text{ MPa} \quad \text{Ans.} \end{aligned}$$

$$\epsilon_x = \epsilon_y = 0$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} \{ \sigma_z - \nu(\sigma_x + \sigma_y) \} = \frac{1}{1 \text{ GPa}} \cdot \{ (-25 \text{ MPa}) - (0.4)(-16.7 - 16.7) \} \\ &= -0.012 \quad \text{Ans.} \end{aligned}$$

b) If the yield stress = 20MPa \rightarrow yield? $(\sigma_y = 20)$

$$\text{Tresca: } \sigma_{y/2} > \tau_{\max} = \text{Max} \left\{ \left| \frac{\sigma_1 - \sigma_2}{2} \right|, \left| \frac{\sigma_2 - \sigma_3}{2} \right|, \left| \frac{\sigma_3 - \sigma_1}{2} \right| \right\}$$

$$\begin{aligned} \sigma_x = -16.7 \text{ MPa} &= \sigma_1 \\ \sigma_y = -16.7 &= \sigma_2 \\ \sigma_z = -25 &= \sigma_3 \end{aligned}$$

$$\text{Max} \left\{ \left| \frac{(-16.7) - (-16.7)}{2} \right|, \left| \frac{-16.7 - (-25)}{2} \right|, \left| \frac{-25 - (-16.7)}{2} \right| \right\}$$

$$= 4.15 = \tau_{\max}$$

$$\sigma_{y/2} > \tau_{\max} ?$$

$$\sigma_{y/2} = 10 \text{ MPa} > 4.15 \text{ MPa} \rightarrow \text{Not Yielding!} \quad \text{Ans.}$$