a solid circular shaft ← bending & torsion

$$\begin{cases} \text{normal stress} : \sigma_{11} \\ \text{shear stress} : \sigma_{12} \end{cases}$$

S_y = normal!

$$SF = \frac{\sigma_{allow}}{\sigma_{current}} = \frac{S_y}{2\tau_{max}} \leftarrow \text{Tresca}$$

$$= \frac{S_y}{\sigma} \leftarrow \text{V.M}$$

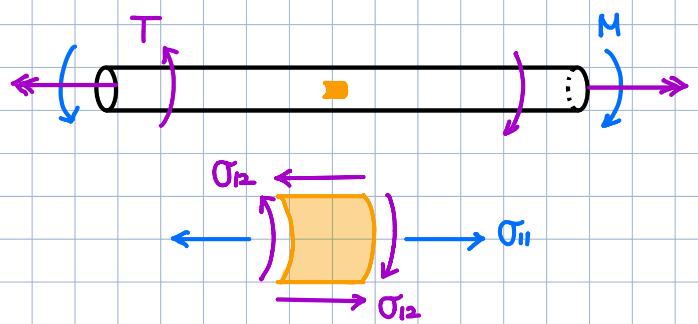
Develope design equation for the shaft → d = function of (S_y, SF, M, T)

Employ the Tresca, von-Mises to determine an expression for SF.

Step 1. Find your current stress conditions

$$\sigma_{11} = \frac{My}{I} = \frac{M \cdot (d/2)}{\left(\frac{\pi d^4}{64}\right)} = \frac{32M}{\pi d^3}$$

$$\sigma_{12} = \frac{Tr}{J} = \frac{T(d/2)}{\left(\frac{\pi d^4}{32}\right)} = \frac{16T}{\pi d^3}$$



Step 2. Find the principal stresses.

$$\sigma_{1,2} = C \pm R \quad \leftarrow \quad C = \frac{\sigma_{11} + \sigma_{22}}{2} = \frac{\frac{32M}{\pi d^3} + 0}{2} = \frac{16M}{\pi d^3}$$

$$R = \sqrt{\left(\frac{\sigma_{11} - \cancel{\sigma_{22}}}{2}\right)^2 + \sigma_{12}^2}$$

$$= \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \cdot \sqrt{M^2 + T^2}$$

$$\sigma_1 = \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{16M}{\pi d^3} - \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

descending order

Step 3. Apply Tresca & von-Mises

Tresca : $$\tau_{max} = \max\left\{ \left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right| \right\}$$

$$= \left|\frac{\sigma_3 - \sigma_1}{2}\right| = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$SF = \frac{S_y}{2\tau_{max}} \longrightarrow \tau_{max} = S_y/2SF = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$\longrightarrow \quad \pi d^3 = 16\sqrt{M^2+T^2}\cdot 2SF/S_y$$

$$d^3 = \frac{32SF}{\pi S_y}\cdot\sqrt{M^2+T^2}$$

$$\therefore\ d = \sqrt[3]{\frac{32SF\cdot\sqrt{M^2+T^2}}{\pi S_y}}$$

Ans.

von-Mises

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\cdot\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

$$= \frac{1}{\sqrt{2}}\cdot\sqrt{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}$$

$$\sigma_{11} = \frac{My}{I} = \frac{M\cdot(d/2)}{\left(\frac{\pi d^4}{64}\right)} = \frac{32M}{\pi d^3}$$

$$\sigma_{12} = \frac{Tr}{J} = \frac{T(d/2)}{\left(\frac{\pi d^4}{32}\right)} = \frac{16T}{\pi d^3}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\cdot\sqrt{\left(\frac{32M}{\pi d^3}-0\right)^2+(0-0)^2+\left(0-\frac{32M}{\pi d^3}\right)^2 + 6\left(\left(\frac{16T}{\pi d^3}\right)^2+0^2+0^2\right)}$$

$$= \frac{1}{\sqrt{2}}\cdot\sqrt{\frac{1024M^2}{\pi^2 d^6}+\frac{1024M^2}{\pi^2 d^6}+6\cdot\frac{256T^2}{\pi^2 d^6}}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\pi d^3}\cdot\sqrt{2048M^2+1536T^2}$$

$$= \frac{1}{\pi d^3}\cdot\sqrt{\underbrace{1024}_{=1024=256\times 4}M^2+\underbrace{768}_{=256\times 3}T^2}$$

$256 = 16^2$

$$= \frac{16}{\pi d^3}\cdot\sqrt{4M^2+3T^2}$$

$$SF = \frac{S_y}{\bar{\sigma}} \longrightarrow \bar{\sigma} = S_y/SF$$

$$= \frac{16}{\pi d^3}\sqrt{4M^2+3T^2}$$

$$\pi d^3 = 16\sqrt{4M^2+3T^2}\cdot SF/S_y$$

$$\therefore d^3 = \frac{16SF}{\pi S_y}\sqrt{4M^2+3T^2}$$

$$d = \sqrt[3]{\frac{16(SF)\cdot\sqrt{4M^2+3T^2}}{\pi S_y}}$$

Ans.

$$\therefore\ d = \sqrt[3]{\frac{32SF\cdot\sqrt{M^2+T^2}}{\pi S_y}}$$

Tresca

$$d = \sqrt[3]{\frac{16(SF)\cdot\sqrt{4M^2+3T^2}}{\pi S_y}}$$

von-Mises