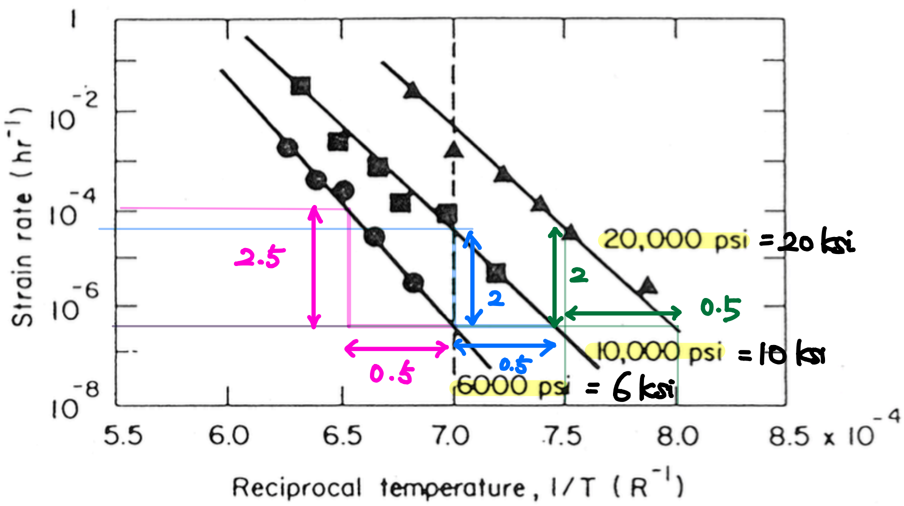
A support rod in a boiler carries a constant tensile stress of 10 ksi. The rod is made from a medium carbon steel with a creep behavior as shown in the figure below and obeys the dimensionless creep equation:

$$\frac{\dot{\varepsilon}_{11}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m e^{\frac{-H}{kT}}$$

where σ_0 is a constant and H the activation energy for creep.

a) What is the activation energy for creep at each stress?
b) Find m, the creep exponent.
c) Find σ_0, the normalizing stress.
d) If this rod elongates more than 10% is must be replaced. What is the lifetime if the boiler operates at 1000°F (°R = °F + 460)?



(a)

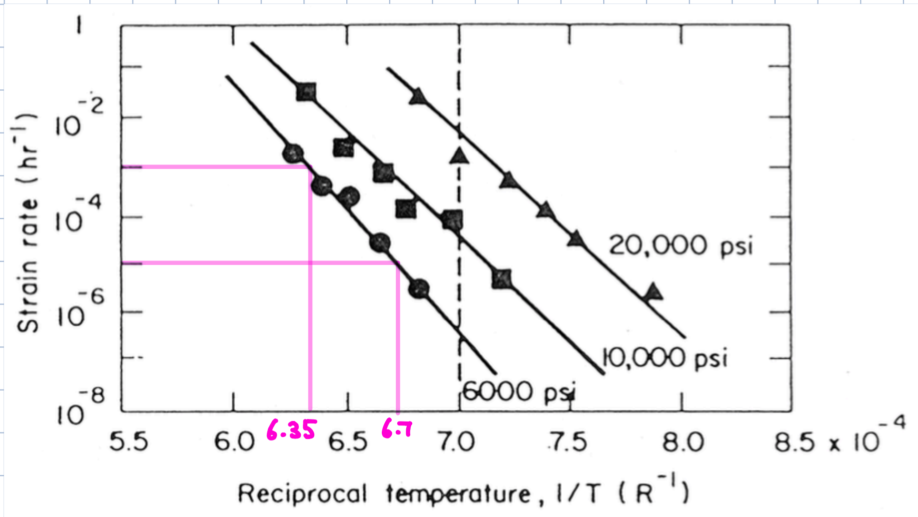
$$\frac{\dot{\varepsilon}_{11}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m e^{-H/kT}$$

$$\ln(\dot{\varepsilon}_{11}/\dot{\varepsilon}_0) = m \cdot \ln(\sigma_{11}/\sigma_0) - H/kT$$ ← σ_{11} = const

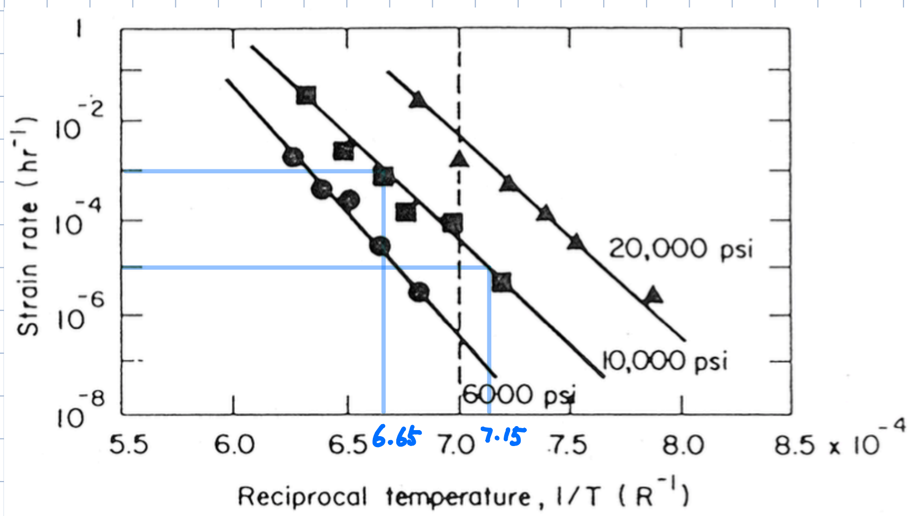
$$H = -k \cdot \left[\frac{\Delta \ln(\dot{\varepsilon}_{11}/\dot{\varepsilon}_0)}{\Delta(1/T)}\right]_{\sigma_{11}=\text{const}}$$

← $k = 1.38 \times 10^{-23}$ J/K $= 6.79 \times 10^{-23}$ in·lb/R

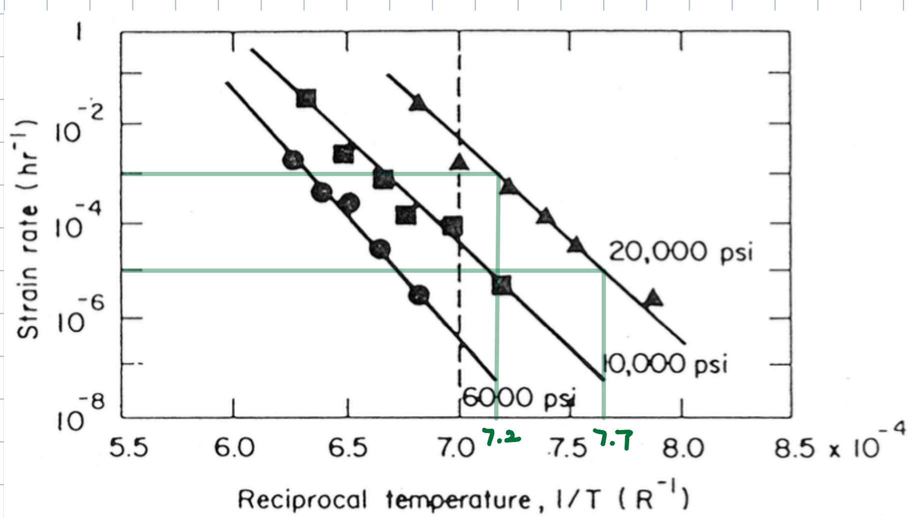
$\dot{\varepsilon}_0 = 1$ (assumption)



• 6 ksi → $H = \left(-6.79\times10^{-23}\ in\cdot lb/R\right)\cdot\dfrac{\ln(10^{-3})-\ln(10^{-5})}{6.35\times10^{-4}-6.7\times10^{-4}} = 8.93\times10^{-18}\ in\cdot lb$ Ans.



• 10 ksi → $H = \left(-6.79\times10^{-23}\ in\cdot lb/R\right)\cdot\dfrac{\ln(10^{-3})-\ln(10^{-5})}{6.65\times10^{-4}-7.15\times10^{-4}} = 6.25\times10^{-18}\ in\cdot lb$ Ans.



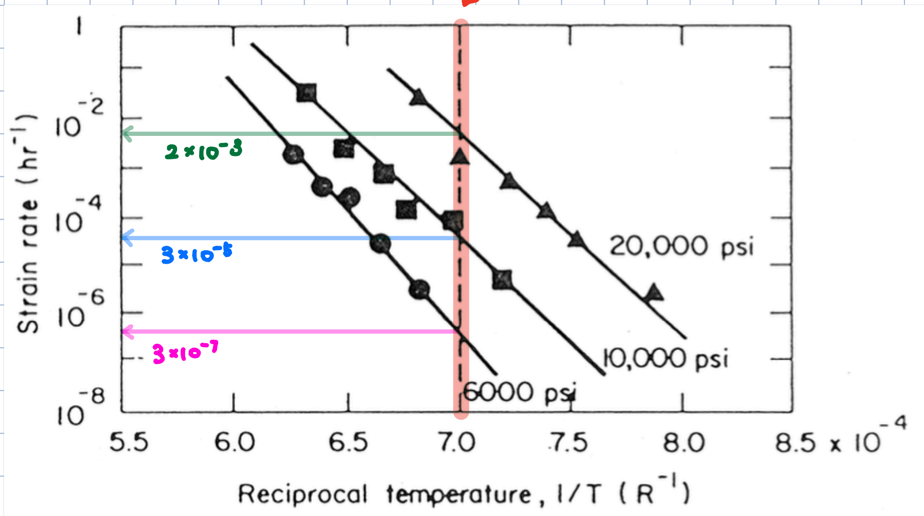
• 20 ksi → $H = \left(-6.79\times10^{-23}\ in\cdot lb/R\right)\cdot\dfrac{\ln(10^{-3})-\ln(10^{-5})}{7.2\times10^{-4}-7.7\times10^{-4}} = 6.25\times10^{-18}\ in\cdot lb$ Ans.

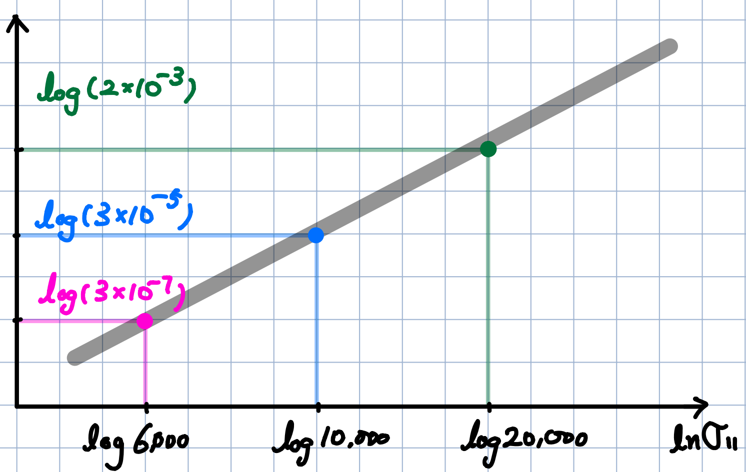
depending on how accurate one reads the graph, these numbers might vary a little.

(b)

$$m = \left[\frac{\Delta \ln(\dot{\varepsilon}_{11})}{\Delta \ln(\sigma_{11})} \right]_{T=const}$$

In this figure, it is convinient to draw a line at $1/T = 7.0 \times 10^{-4}\ °R^{-1}$





$$m_1 = \frac{\log(3\times10^{-7}) - \log(3\times10^{-5})}{\log(6{,}000) - \log(10{,}000)} = 9.015$$

$$m_2 = \frac{\log(3\times10^{-7}) - \log(2\times10^{-3})}{\log(6{,}000) - \log(20{,}000)} = 7.313$$

$$m_3 = \frac{\log(3\times10^{-5}) - \log(2\times10^{-3})}{\log(10{,}000) - \log(20{,}000)} = 6.059$$

∴ $m \sim 7.462$ (an average value) Ans.

depending on how accurate one reads the graph, these numbers might vary a little.

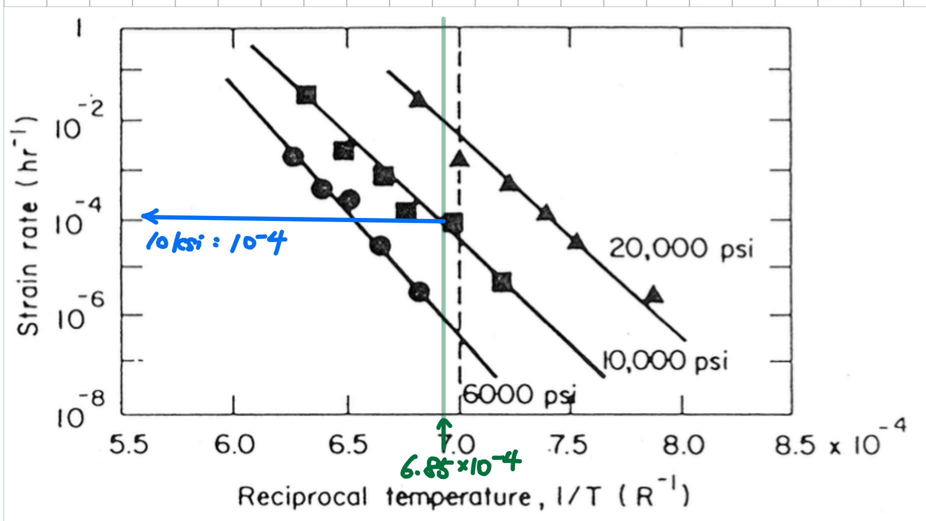
(c) $\dfrac{\dot{\varepsilon}_{11}}{\dot{\varepsilon}_0} = \left(\dfrac{\sigma_{11}}{\sigma_0}\right)^m e^{-H/kT}$

H, m : knowns

σ_0 : Unknown

$$\left(\dot{\varepsilon}_{11}/\dot{\varepsilon}_0\right)^{1/m} = \sigma_{11}/\sigma_0 \cdot e^{-H/mkT}$$

$$\therefore \sigma_0 = \sigma_{11} \cdot \left(\dot{\varepsilon}_0/\dot{\varepsilon}_{11}\right)^{1/m} \cdot e^{-H/mkT}$$



$\dot{\varepsilon}_0 = 1\ hr^{-1}$

$\dot{\varepsilon}_{11} = 10^{-4}\ hr^{-1}$

$\sigma_{11} = 10\ ksi = 10{,}000\ psi$

$T = 1460\ R$

$H = 6.25 \times 10^{-18}\ in \cdot lb$ (when 10 ksi)

$m \sim 7.462$

$k = 1.38 \times 10^{-23}\ J/K$
$= 6.79 \times 10^{-23}\ in \cdot lb/R$

$$\therefore \sigma_0 = (10{,}000\ psi) \cdot \left(\frac{1\ hr^{-1}}{10^{-4}\ hr^{-1}}\right)^{1/7.462} \cdot \exp\left\{\frac{-6.25\times10^{-18}\ in \cdot lb}{(7.462)(6.79\times10^{-23}\ in \cdot lb/R)(1460\ R)}\right\}$$

$= 7.35\ psi$ Ans.

(d) $\dot{\varepsilon}_{11} = $ constant , $\varepsilon_{11} = 0.1$

$\dfrac{\dot{\varepsilon}_{11}}{\dot{\varepsilon}_0} = \left(\dfrac{\sigma_{11}}{\sigma_0}\right)^m e^{-H/kT}$

$$\dot{\varepsilon}_{11} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m e^{-H/kT} = \left(\frac{10{,}000\ psi}{7.35\ psi}\right)^{7.462} \exp\left\{\frac{-6.25\times10^{-18}\ in \cdot lb}{(6.79\times10^{-23}\ in \cdot lb/R)(1460\ R)}\right\}$$

$= 0.0001007496$ ← $\dot{\varepsilon}_{11} = \varepsilon_{11}/t$

$$t = \varepsilon_{11}/\dot{\varepsilon}_{11} = \frac{0.1}{0.0001007496} = 992.559 \sim 1{,}000\ hours$$ Ans.

Creep life is 1000 hours