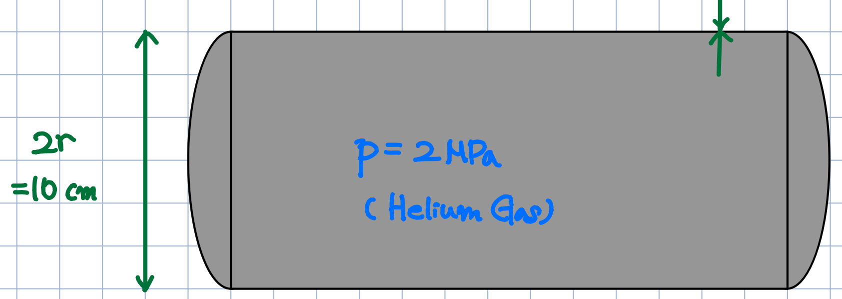
A nuclear pressure vessel, made of annealed Zircaloy 2 alloy, is pressurized with helium gas at 2 MPa for six weeks at 650 F. It is then subjected to an increase in internal pressure to 5 MPa for an additional four weeks at 625 F. If the initial wall thickness of the tube was $t = 5$ mm and an outer diameter of $D = 10$ cm, calculate the final dimensions of the pressure tube after the 10 weeks period (length remains unchanged).

To solve, you may neglect elastic and primary creep strains and assume that for the temperature range of interest, Zircaloy 2 follows a constitutive law:

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} = \left(\frac{\sigma}{\sigma_o}\right)^{16} e^{-\frac{H}{kT}} \text{ hr}^{-1} \qquad \text{where } \sigma_o = 40 \text{ MPa}, \ \dot{\varepsilon}_0 = 1 \text{ hr}^{-1} \text{ and } H = 1.2 \times 10^{-19} \text{ J}$$

initial thickness, $t = 5$ mm



6 weeks at 650°F (p=2MPa)

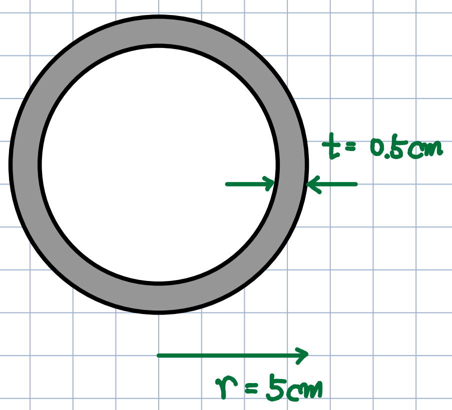
↓

4 weeks at 625°F (p=5MPa)

A nuclear pressure vessel
(Zircaloy 2 alloy)

Calculate the final dimension of the tube after the 10 weeks

0. Thin-walled approximation



$$r/t = \frac{5\text{ cm}}{0.5\text{ cm}} = 10 \gg 1$$

thin-walled approximation is valid Ans.

1. Stresses (Equilibrium)

$\boxed{\sigma_{\theta\theta} = Pr/t}$, $\sigma_{rr} = 0$ ($\because$ thin-walled approximation)

$\sigma_{zz} = Pr/2t$ (this will be ignored as it's a full factor of 2 less than $\sigma_{\theta\theta}$.)

2. Strains (Compatibility)

$\boxed{\varepsilon_{\theta\theta} = \ln(r/r_o)}$, $\boxed{\varepsilon_{rr} = \ln t/t_o}$, $\varepsilon_{zz} = 0$ ($\because$ no elongation in the z-direction)

3. Constitutive Law (Stress $\longleftrightarrow$ Strain)

Constitutive Law : $\dot{\varepsilon}/\dot{\varepsilon}_0 = (\sigma/\sigma_0)^{16} e^{-H/kT}$ (hr^{-1}) $\longleftarrow$

$\sigma_0 = 40\,MPa$

$\dot{\varepsilon}_0 = 1\ hr^{-1}$

$H = 1.2\times10^{-19}\,J$

$k = 1.38\times10^{-23}\,J/K$

$$\boxed{\sigma_{\theta\theta} = Pr/t} \longleftrightarrow \boxed{\varepsilon_{\theta\theta} = \ln(r/r_0)}$$

$$\boxed{\dot{\varepsilon}_{\theta\theta} = (\sigma_{\theta\theta}/\sigma_0)^{16} \cdot \exp\{-H/kT\}}$$

• 6 weeks at 650°F $\longrightarrow$ $\begin{cases} 6\text{ week} = 1008\text{ hours} \\ 650°F = 616.5\,K \end{cases}$

$$\dot{\varepsilon}_{\theta\theta} = \left\{\frac{\frac{(2MPa)\cdot(0.05m)}{(0.005m)}}{40MPa}\right\}^{16} \cdot \exp\left\{\frac{-1.2\times10^{-19}\,J}{(1.38\times10^{-23}\,J/K)(616.5K)}\right\} = \underline{1.1425\times10^{-11}\ hour^{-1}}$$

$$\varepsilon_{\theta\theta} = \dot{\varepsilon}_{\theta\theta}\,t = (1.1425\times10^{-11}\ hour^{-1})(1008\ hours) = \underline{1.1516\times10^{-8}}$$ Ans.

• 4 weeks at 625°F $\longrightarrow$ $\begin{cases} 4\text{ week} = 672\text{ hours} \\ 625°F = 602.6\,K \end{cases}$

$$\dot{\varepsilon}_{\theta\theta} = \left\{\frac{\frac{(5MPa)\cdot(0.05m)}{(0.005m)}}{40MPa}\right\}^{16} \cdot \exp\left\{\frac{-1.2\times10^{-19}\,J}{(1.38\times10^{-23}\,J/K)(602.6K)}\right\} = \underline{1.921\times10^{-5}\ hour^{-1}}$$

$$\varepsilon_{\theta\theta} = \dot{\varepsilon}_{\theta\theta}\,t = (1.921\times10^{-5}\ hour^{-1})(672\ hours) = \underline{1.291\times10^{-2}}$$ Ans.

$\therefore$ Total Strain : $\varepsilon_{\theta\theta, total} = (1.1516\times10^{-8}) + (1.291\times10^{-2}) \approx \underline{1.291\times10^{-2}}$ Ans.

$\varepsilon_{\theta\theta} = \ln r/r_o \longrightarrow r/r_o = e^{\varepsilon_{\theta\theta}} \longrightarrow r = r_o \exp\{\varepsilon_{\theta\theta}\} = (5cm)\cdot\exp\{1.291\times10^{-2}\}$

$= 5.065\,cm$

$\therefore \underline{D = 10.13\,cm}$ Ans.

$\varepsilon = \varepsilon_{rr} + \varepsilon_{\theta\theta} + \cancel{\varepsilon_{zz}} = 0$ (plasticity) $\because \Delta V/V = 0$

0

$\therefore \varepsilon_{rr} = -\varepsilon_{\theta\theta}$

$\therefore \varepsilon_{rr} = -\varepsilon_{\theta\theta} = -\ln r/r_o = -\ln\left(\frac{5.065\,cm}{5\,cm}\right) = -0.0129$

$\boxed{\varepsilon_{rr} = \ln t/t_o} = -0.0129$

$t/t_o = e^{-0.0129}$

$\therefore t = (t_o)\cdot e^{-0.0129}$

$= (5\,mm)\cdot e^{-0.0129}$

$= \underline{4.936\,mm}$ Ans.