Plastic Zone Size for Plane Stress

the stress intensity factor

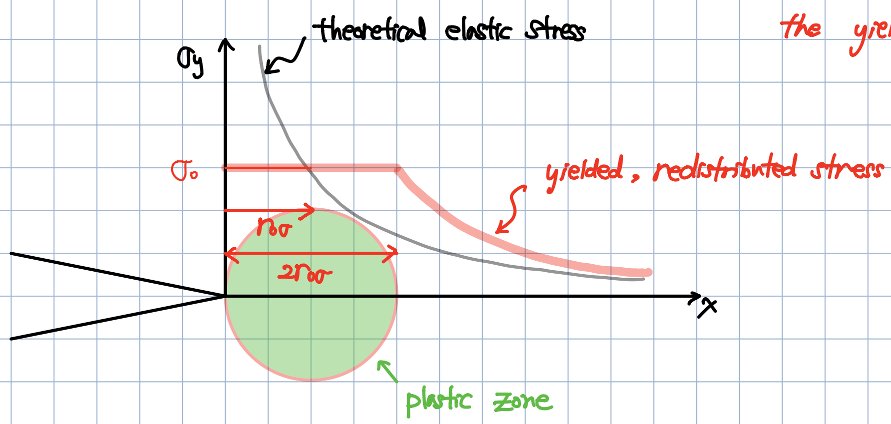
$$K = FS\sqrt{\pi a} \xrightarrow[F \approx 1]{} S = \frac{K}{\sqrt{\pi a}} \longrightarrow \begin{cases} \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \cdot \left\{ 1 - \sin\theta \sin\frac{3\theta}{2} \right\} \\ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \cdot \left\{ 1 + \sin\theta \sin\frac{3\theta}{2} \right\} \\ \sigma_z = 0 \quad (\because \text{plane stress}) \end{cases}$$

$\theta = 0 \rightarrow$ σ_x, σ_y, and σ_z are principal normal stresses $\rightarrow$ yielding occurs at $\underline{\sigma_x = \sigma_y = \sigma_o}$ ★★★

$$\sigma_x = \sigma_y = \frac{K}{\sqrt{2\pi r}} = \sigma_o$$

$$\therefore \sigma_o = \frac{K}{\sqrt{2\pi r_{o\sigma}}} \rightarrow r_{o\sigma} = \frac{1}{2\pi} \cdot \left(\frac{K}{\sigma_o}\right)^2 \rightarrow \underline{2r_{o\sigma} = \frac{1}{\pi} \cdot \left(\frac{K}{\sigma_o}\right)^2}$$ Ans.

This is the distance ahead of the crack tip where the elastic stress distribution exceeds the yield criterion for plane stress.



Yielding within the plastic zone

The stresses are lower than the values from the elastic stress-field equations.

The yielded material offer less resistance than expected.

Large deformations occur.

$\therefore r_{o\sigma} \rightarrow 2r_{o\sigma}$

Plastic Zone Size for Plane Strain

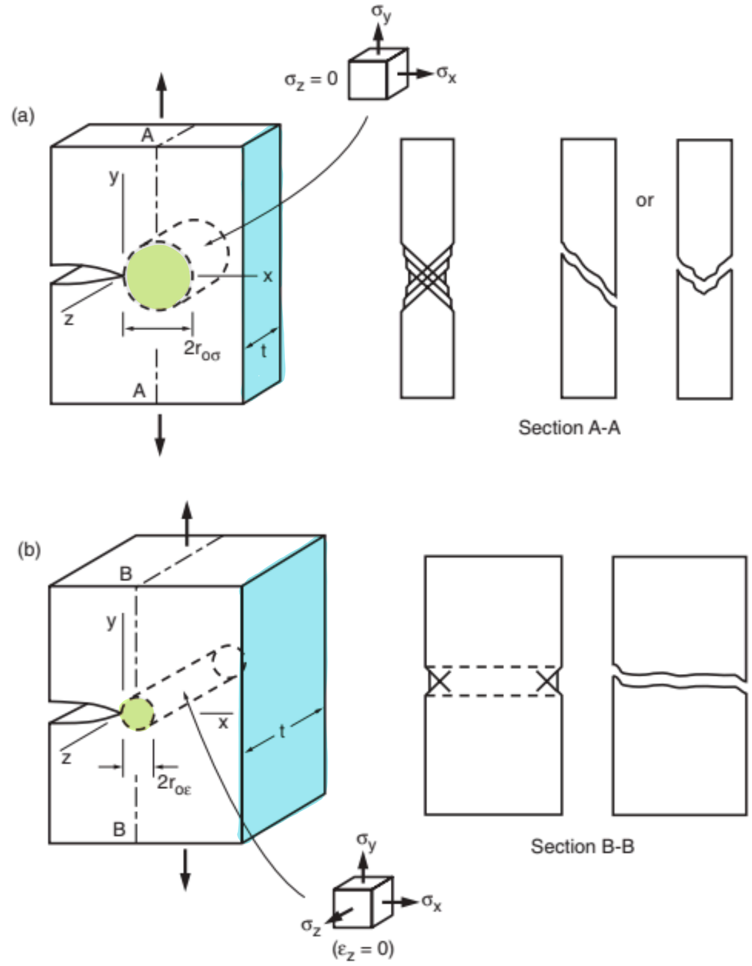


Figure 8.44 Plastic zone, stress state, and fracture mode for (a) plane stress and (b) plane strain.

Why $2r_{o\varepsilon} < 2r_{o\sigma}$???

$\varepsilon_z = 0$, $\sigma_x = \sigma_y = \dfrac{K_I}{\sqrt{2\pi r}}$ $\longrightarrow$ $\cancel{\varepsilon_z} = \frac{1}{E}\{\sigma_z - \nu(\sigma_x+\sigma_y)\}$

$\because$ plane strain

$\therefore \sigma_z = \nu(\sigma_x+\sigma_y) = 2\nu\sigma_x$

Applying von-Mises :

$$\sigma_0 = \frac{1}{\sqrt{2}}\cdot\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}^{\frac{1}{2}} \longleftarrow$$

$\theta = 0$;

$\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = 2\nu\sigma_x$

$$= \frac{1}{\sqrt{2}}\cdot\{\cancel{(\sigma_x-\sigma_y)^2}+(\underbrace{\sigma_y}_{=\sigma_x}-2\nu\sigma_x)^2+(2\nu\sigma_x-\sigma_x)^2\}^{\frac{1}{2}}$$

$\because \sigma_x = \sigma_y$

$$= \frac{1}{\cancel{\sqrt{2}}}\cdot\sqrt{\cancel{2}\cdot\sigma_x^2\cdot(1-2\nu)^2}$$

$$= (1-2\nu)\cdot\sigma_x$$

$$\therefore \sigma_x = \frac{\sigma_0}{1-2\nu}$$

if we apply ν as a typical value $\nu = 0.3$, $\sigma_x = \sigma_y = \dfrac{\sigma_0}{1-2\times0.3} = \dfrac{\sigma_0}{0.4} = 2.5\sigma_0$ Ans.

$$\boxed{t,\ a,\ (b-a),\ h \geq 2.5\left(\frac{K}{\sigma_0}\right)^2}$$ (plane strain)

However, the more refined estimate by G.R. Irwin suggests that
this effect is somewhat smaller
with yielding around $\sigma_y = \sqrt{3}\,\sigma_0 \approx 1.7\sigma_0$ Ans.

$$\therefore \sigma_x = \sqrt{3}\cdot\sigma_0 = \frac{K}{\sqrt{2\pi r_{0\varepsilon}}}$$

$$\therefore 2r_{0\varepsilon} = \frac{1}{3\pi}\cdot\left(\frac{K}{\sigma_0}\right)^2 \quad \longleftrightarrow \quad 2r_{0\sigma} = \frac{1}{\pi}\cdot\left(\frac{K}{\sigma_0}\right)^2$$

Ans.

plane strain　　　　plane stress

Plasticity Limitations on LEFM

Practically, $8r_o$ is generally considered to be efficient.

4 times the plastic zone size
($2r_{o\sigma}$, $2r_{o\varepsilon}$)

$$2r_{o\sigma} = \frac{1}{\pi} \cdot \left(\frac{K}{\sigma_o}\right)^2$$

$$\downarrow$$

$$8r_{o\sigma} = \frac{4}{\pi} \cdot \left(\frac{K}{\sigma_o}\right)^2 \longrightarrow$$

$$\boxed{a,\ (b-a),\ h \ \geq \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2 \quad \text{(LEFM applicable)}}$$

$$\boxed{t,\ a,\ (b-a),\ h \ \geq 2.5\left(\frac{K}{\sigma_o}\right)^2 \quad \text{(plane strain)}}$$

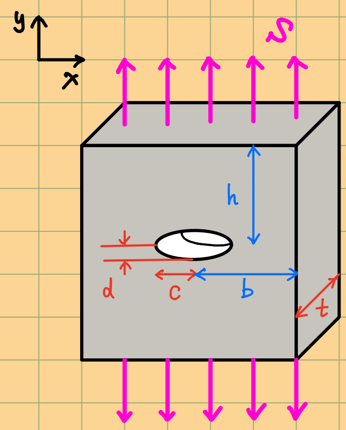
Example>

- 2014-T651 Al : $\sigma_0 = 415\ MPa$

(a) $a = 10\ mm$ → LEFM is valid?
the plastic zone size?

- $t = 5mm$, $b = 50mm$, $K = 17.7 MPa\sqrt{m}$

A center-cracked plate



$b = 50\ mm$
$t = 5\ mm$
large h

$P = 50kN$

tip radius, $\rho = d^2/c$

$$\boxed{a,\ (b-a),\ h \geq \frac{4}{\pi}\left(\frac{K}{\sigma_0}\right)^2 \quad \text{(LEFM applicable)}}$$

$$\boxed{t,\ a,\ (b-a),\ h \geq 2.5\left(\frac{K}{\sigma_0}\right)^2 \quad \text{(plane strain)}}$$

$$5,\ 10,\ 40,\ \text{large } h \geq (2.5)\cdot\left(\frac{17.7 MPa\sqrt{m}}{415MPa}\right)^2$$

$$= 0.0045m = \underline{4.5mm}$$ Ans.

plane strain & LEFM are applicable.

$$\boxed{\therefore\ 2r_{o\varepsilon} = \frac{1}{3\pi}\cdot\left(\frac{K}{\sigma_0}\right)^2} = \left(\frac{1}{3\pi}\right)\cdot\left(\frac{17.7 MPa\sqrt{m}}{415MPa}\right)^2 = \underline{0.19\ mm}$$ Ans.

b) $a_c = 16.3\ mm$ → ?

- $t = 5mm$, $b = 50mm$, $K = 24\ MPa\sqrt{m}$

$$\boxed{t,\ a,\ (b-a),\ h \geq 2.5\left(\frac{K}{\sigma_0}\right)^2 \quad \text{(plane strain)}}$$

$$5,\ 16.3,\ 33.7,\ \text{large } h \geq (2.5)\cdot\left(\frac{24 MPa\sqrt{m}}{415MPa}\right)^2 = \underline{8.4mm}$$ Ans.

plane strain is NOT applicable

$$\boxed{a,\ (b-a),\ h \geq \frac{4}{\pi}\left(\frac{K}{\sigma_0}\right)^2 \quad \text{(LEFM applicable)}}$$

$$16.3,\ 33.7,\ \text{large } h \geq (4/\pi)\cdot\left(\frac{24 MPa\sqrt{m}}{415MPa}\right)^2 = \underline{4.3\ mm}$$ Ans.

LEFM is applicable

$$\boxed{\therefore\ 2r_{o\sigma} = \frac{1}{\pi}\cdot\left(\frac{K}{\sigma_0}\right)^2} = \left(\frac{1}{\pi}\right)\cdot\left(\frac{24 MPa\sqrt{m}}{415MPa}\right)^2 = \underline{1.06\ mm}$$ Ans.