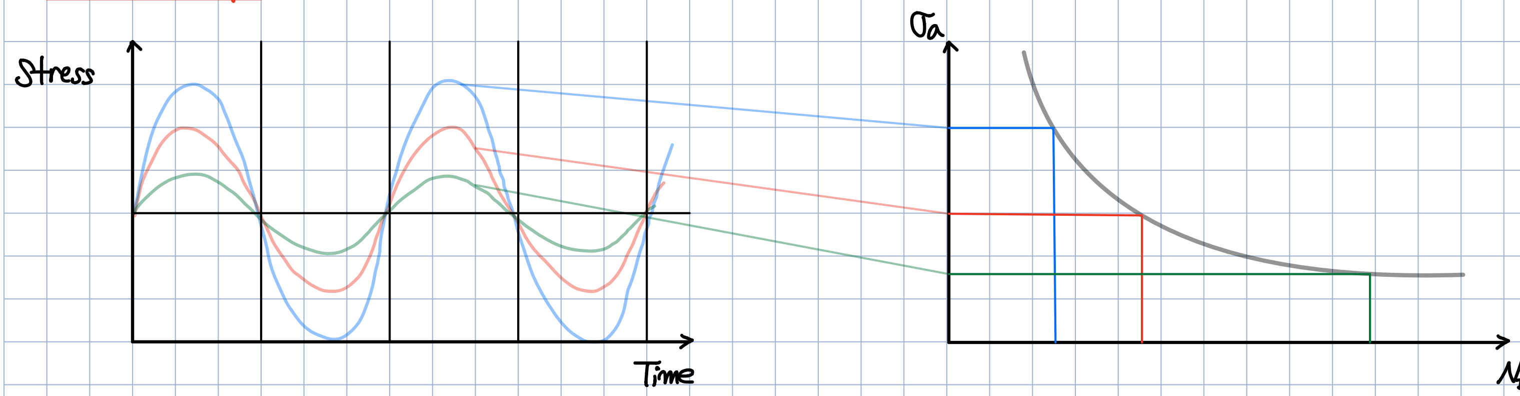
Fatigue of Engineering Materials

Why is cyclic loading so problematic?
How do we design against fatigue failures?

1. Total Life : the component is initially free of any flaws that are sufficiently sized for growth or ideally that the component is defect-free.
   - S-N plot
   - Basquin
   - Goodman
   - Miner's Rule

2. Defect-tolerant : the component has an existing initial flaw capable of propagating.
   - da/dN curve

1. Total Life



Basquin Equation : $\sigma_a = \sigma_f'(N_f)^b$

σ_a = stress amplitude

σ_f' : material property

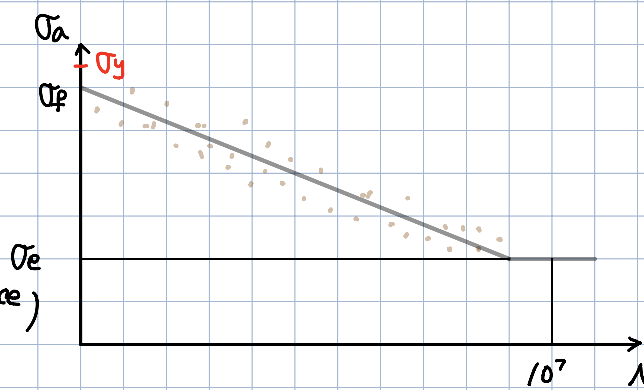
$$\log \sigma_a = \log(\sigma_f' \cdot N_f^b)$$
$$= \log \sigma_f' + b \cdot \log N_f$$

$$\begin{aligned} &\log \sigma_{a,1} = \cancel{\log \sigma_f'} + b \cdot \log N_{f,1} \\ - &\log \sigma_{a,2} = \cancel{\log \sigma_f'} + b \cdot \log N_{f,2} \end{aligned}$$

$$\log \sigma_{a,1} - \log \sigma_{a,2} = b \cdot (\log N_{f,1} - \log N_{f,2})$$

$$\therefore b = \frac{\log \sigma_{a,1} - \log \sigma_{a,2}}{\log N_{f,1} - \log N_{f,2}}$$

Ans.



(= the endurance limit)

S-N plot on linear-logarithmic scale

σ_e could be affected by
- surface finish
- stress concentration
- heat-treatment
- environment
- component design

- σ_e : the endurance limit at zero mean stress
  (An endurance limit is generally defined as the cyclic stress level that enables infinite life in the material)

- σ_f : the failure stress that is defined as the ultimate strength of the material, σ_{UTS}.

Basquin Equation : $\sigma_a = \sigma_f'(N_f)^b$ $\longleftrightarrow$ $\sigma_a = \sigma_f'(2N_f)^b$

Example >

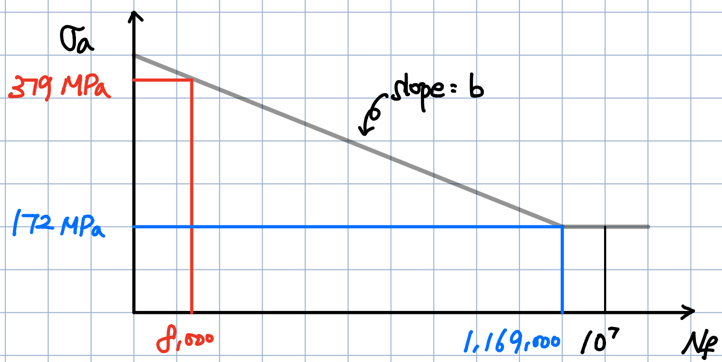
Unnotched specimens

frequency : 20 Hz

Stress Amplitude, σ_a (MPa)	Cycle to failures (N_f)
379	8,000
345	13,000
276	53,000
207	306,000
172	1,169,000

a) Determine the coefficients and basic form of the Basquin equation

b) If the stress amplitude is 200 MPa, how many cycle will it last?



$\boxed{\text{Basquin Equation : } \sigma_a = \sigma_f'(N_f)^b}$

$$\log \sigma_a = \log(\sigma_f' \cdot N_f^b)$$
$$= \log \sigma_f' + b \cdot \log N_f$$

$$\log \sigma_{a,1} = \cancel{\log \sigma_f'} + b \cdot \log N_{f,1}$$
$$- \ \log \sigma_{a,2} = \cancel{\log \sigma_f'} + b \cdot \log N_{f,2}$$

$$\log \sigma_{a,1} - \log \sigma_{a,2} = b \cdot (\log N_{f,1} - \log N_{f,2})$$

$$\therefore b = \frac{\log \sigma_{a,1} - \log \sigma_{a,2}}{\log N_{f,1} - \log N_{f,2}}$$

$$b = \frac{\log 379 - \log 172}{\log 8,000 - \log 1,169,000} = \underline{-0.159}$$

$$(379\,\text{MPa}) = \sigma_f' \cdot (8,000)^{-0.159}$$

$$\therefore \underline{\sigma_f' = 1,582\ \text{MPa}}$$

$$\therefore \underline{\sigma_a = (1,582) \cdot N_f^{-0.159}} \quad \text{Ans.}$$

$$N_f = (\sigma_a / \sigma_f')^{1/b} \longleftarrow \sigma_a = 200\,\text{MPa}$$

$$= \left(\frac{200\,\text{MPa}}{1582\,\text{MPa}}\right)^{1/-0.159} = \underline{445,600 \text{ cycles}} \quad \text{Ans.}$$

Basquin Equation : $\sigma_a = \sigma_f'(N_f)^b$

σ_a = stress amplitude

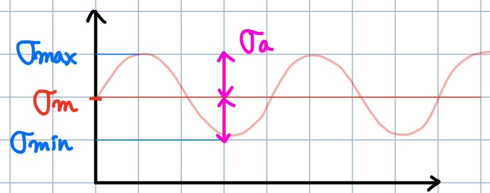
σ_f' : material property

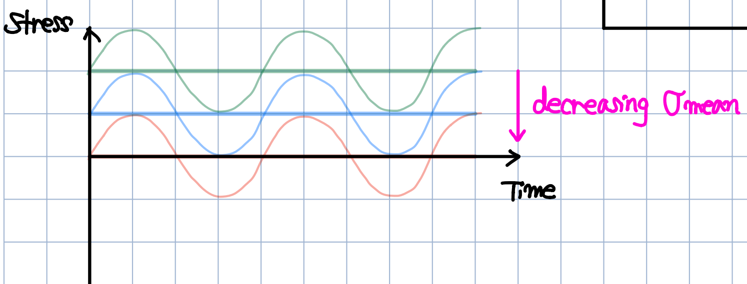
The use of the Basquin equation assumes that the mean stress is zero. $\longrightarrow$ fully reversed loading

If the mean stress is not zero, then these effects must be considered.

As the mean stress of a fatigue cycle is increased,
the number of cycles to failure and the endurance limit is decreased substantially.

The Goodman Relationship





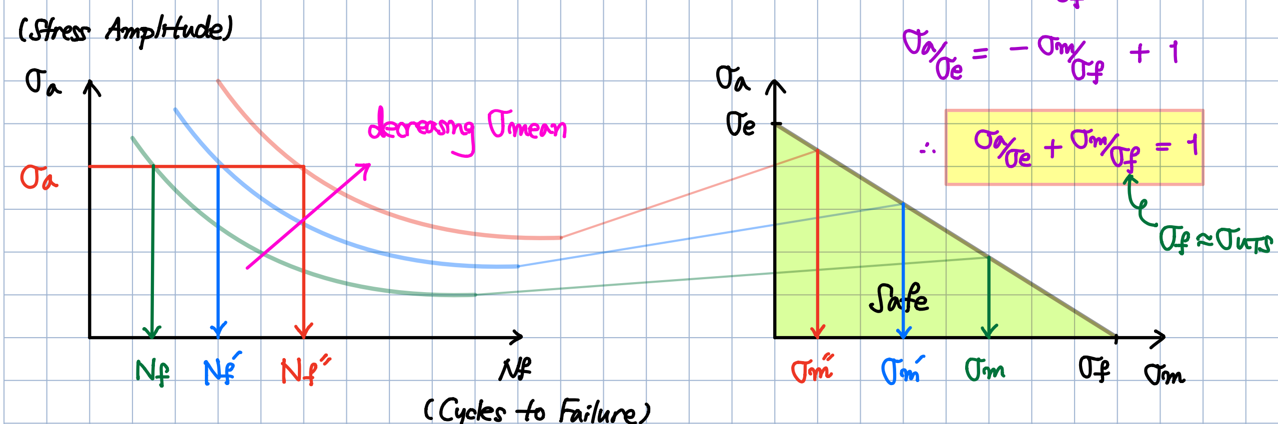
- assumptions
  - : The endurance limit decreases proportionately with increasing mean stress.

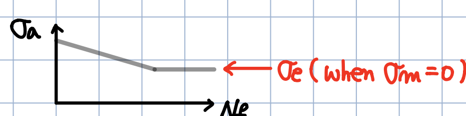
$$\sigma_a = -\frac{\sigma_e}{\sigma_f}\cdot\sigma_m + \sigma_e$$

$$\frac{\sigma_a}{\sigma_e} = -\frac{\sigma_m}{\sigma_f} + 1$$

$$\therefore \boxed{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_f} = 1}$$

$\sigma_f \approx \sigma_{UTS}$





$$\boxed{\sigma_a = \sigma_e\cdot\left(1-\frac{\sigma_m}{\sigma_f}\right)}$$

$$= \sigma_a\big|_{\sigma_m=0}\cdot\left(1-\frac{\sigma_m}{\sigma_f}\right) \longleftarrow \text{Basquin : } \sigma_a = \sigma_f\cdot N_f^{\,b}$$

$$= \sigma_f\cdot N_f^{\,b}\cdot\left(1-\frac{\sigma_m}{\sigma_f}\right)$$

$$= \sigma_f\cdot N_f^{\,b} - \sigma_m\cdot N_f^{\,b}$$

$$\therefore \boxed{\sigma_a = (\sigma_f-\sigma_m)\cdot N_f^{\,b}}$$

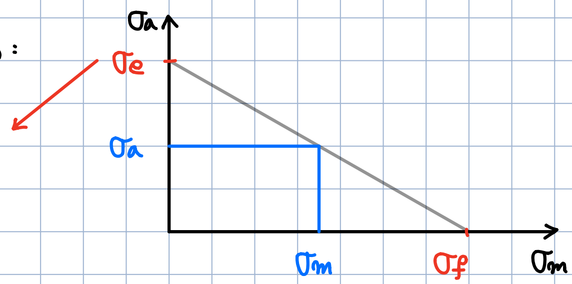
Example >

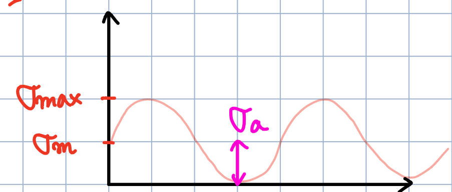
endurance limit, $\sigma_e = 547$ MPa
fracture strength, $\sigma_f = 1050$ MPa

- How is the maximum cyclic stress allowable, when stress ranges from 0 to the max $(R=0)$?
  $(\sigma_a = ?)$

Goodman Relationship :

$\sigma_a \big|_{\sigma_m = 0}$





$\sigma_m = \dfrac{\sigma_{max}}{2}$, $\sigma_a = \sigma_m$ ★★

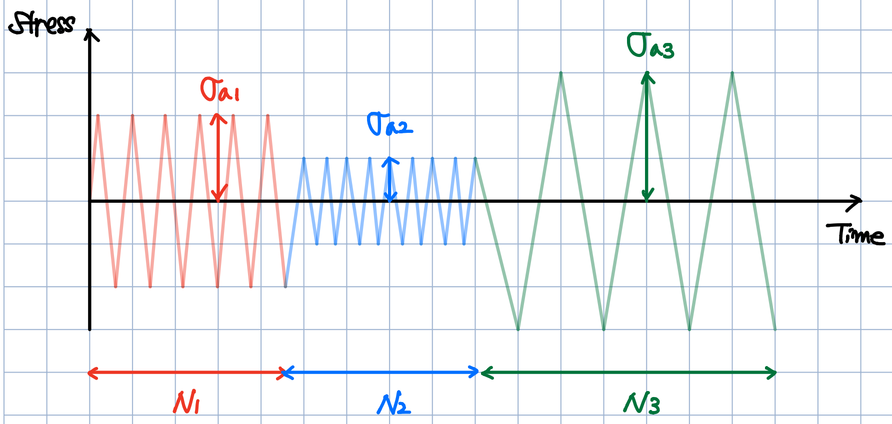
$\boxed{\sigma_a/\sigma_e + \sigma_m/\sigma_f = 1}$ $\rightarrow$ $\dfrac{\sigma_a}{547} + \dfrac{\sigma_m}{1050} = 1$ $\leftarrow$ $\sigma_m = \sigma_a$

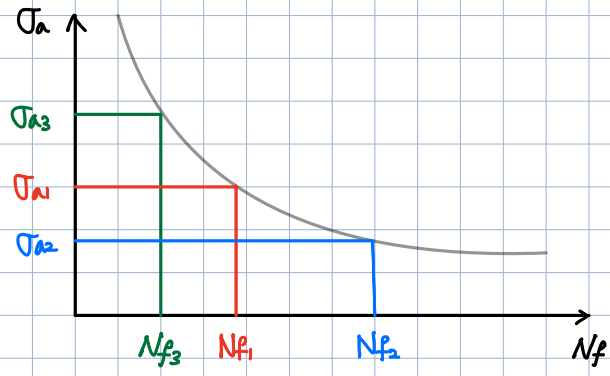
$$\sigma_a \left(\frac{1}{547} + \frac{1}{1050} \right) = 1$$

$\therefore \sigma_a = 359$ MPa Ans.

Miner's Rule : Cumulative Damage

variable amplitude loading and fractional life calculations





$$\boxed{\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \cdots = 1}$$

Example >

Unnotched specimens

σ_a (MPa)	N_f
379	8,000
345	13,000
276	53,000
207	306,000
172	1,169,000

Stress Amplitude σ_i (MPa)	Cycles N_i
250	1
200	3
100 (comp)	100
100 (tensile)	100

the specimen is expected to undergo the following load cycles each hour.

$$\boxed{\text{Basquin Equation: } \sigma_a = \sigma_f'(2N_f)^b}$$

$$\log \sigma_a = \log \sigma_f' + b \cdot \log(2N_f)$$

$$\begin{aligned} &\log \sigma_{a1} = \cancel{\log \sigma_f'} + b \cdot \log(2N_{f1}) \\ - \quad &\log \sigma_{a2} = \cancel{\log \sigma_f'} + b \cdot \log(2N_{f2}) \end{aligned}$$

$$b = \frac{\log \sigma_{a1} - \log \sigma_{a2}}{\log(2N_{f1}) - \log(2N_{f2})} = \frac{\log(379) - \log(172)}{\log(2 \cdot 8000) - \log(2 \cdot 1169000)} = \underline{-0.159}$$

$$379 \text{ MPa} = \sigma_f' \cdot (2 \times 8000)^{-0.159} \longrightarrow \underline{\sigma_f' = 1766 \text{ MPa}}$$

$$\boxed{\sigma_a = 1766 \cdot (2N_f)^{-0.159}} \longrightarrow 2N_f = \left(\frac{\sigma_a}{1766}\right)^{1/-0.159} \longrightarrow N_f = \frac{1}{2} \cdot \left(\frac{\sigma_a}{1766}\right)^{1/-0.159}$$

Stress Amplitude σ_i (MPa)	Cycles N_i	Cycles to failure (N_f)	Damage ($d_i = N_i/N_f$)
250	1	1.0937 e5	9.14 e-6
200	3	4.4505 e5	2.25 e-6
100 (comp)	100	3.4808 e7	2.87 e-8
100 (tensile)	100	3.4808 e7	2.87 e-8

$$D = \Sigma d_i = 1.145\text{e-}5$$

Blocks to failure, $B_f = 1/D = 1/1.145\text{e-}5$ = 8.74e4 blocks

= 8.74e4 hours ← 365 days/yr × 24 hours/day = 8760 hours/yr

~ 9.97 years Ans.

Example >

The variations in load are

$$\begin{cases} 1 \text{ reversal} : 0 \sim 800 \text{ MPa} \\ 10 \text{ reversals} : 220 \sim 800 \text{ MPa} \end{cases}$$

Basquine : $\sigma_a = \sigma_f' N_f^b$

a) Determine σ_f' and b

b) Using Miner's Rule, estimate the number of loading blocks

σ_a (MPa)	N_f (cycles)
948	222
834	992
703	6004
631	14130
579	43860
524	132150

$$b = \frac{\log(948) - \log(524)}{\log(222) - \log(132150)} = \underline{-0.0928} \quad \text{Ans.}$$

$$\sigma_f' = \sigma_a / N_f^b = \frac{948}{(222)^{-0.0928}} = \underline{1565 \text{ MPa}} \quad \text{Ans.}$$

σ_a (MPa)	σ_m	Cycles at this amplitude, N_i	N_f	Damage, $d_i = N_i/N_f$
400	400	1	100,662	9.93 e-6
290	510	10	1,105,834	9.04 e-6

$$\sigma_a = (\sigma_f' - \sigma_m) \cdot N_f^b \longrightarrow N_f = \left(\frac{\sigma_a}{\sigma_f' - \sigma_m}\right)^{1/b} = \left(\frac{400}{1565 - 400}\right)^{1/-0.0928} = 100,662$$

$$= \left(\frac{290}{1565 - 510}\right)^{1/-0.0928} = 1,105,834$$

$D = \Sigma d_i = 1.89\text{e-}5$

Blocks to failure, $B_f = 1/D = 1/1.89\text{e-}5 = \underline{52,706 \text{ blocks}}$ Ans.

$= 52,706$ hours