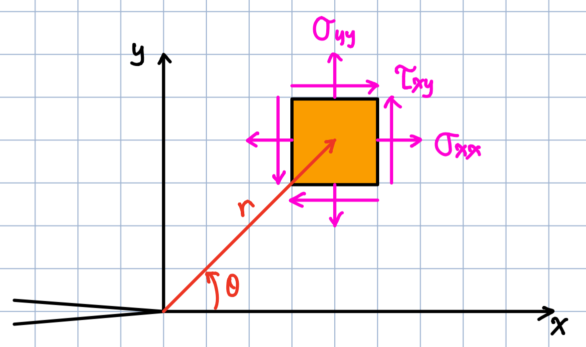
Defect-tolerant philosophy

The structural components are intrinsically flawed and the fatigue life is based on propagation of an initial flaw to a critical size

LEFM → The stress intensity factor, K
↳ the parameter to describe the stress/strains/displacements ahead of the crack-tip.



Mode I

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \cdot \left(1 - \sin\frac{\theta}{2} \cdot \sin\frac{3\theta}{2}\right)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \cdot \left(1 + \sin\frac{\theta}{2} \cdot \sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \cdot \left(\sin\frac{\theta}{2} \cdot \sin\frac{3\theta}{2}\right) + \cdots$$

• Concept : The velocity of a moving fatigue crack subjected to constant stress amplitude loading is determined from the change in crack length, a, as a function of the number of loading cycles, N.

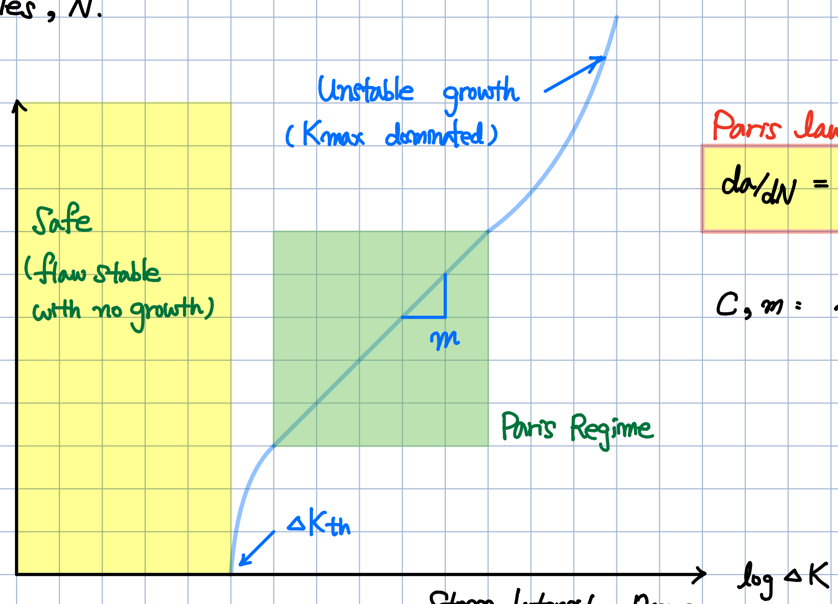
log da/dN

Fatigue-Crack Growth Rate
da/dN (mm/cycle)

a : crack length

N : the number of loading cycles

da/dN : (The velocity)
The fatigue crack growth per loading cycle.



Paris law :

$$\boxed{da/dN = C \cdot \Delta K^m}$$

C, m : material constants

ΔK : the stress intensity factor range
$\Delta K = K_{max} - K_{min}$

$K = f(a/w)\,\sigma^{\infty}\sqrt{\pi a}$ ←
1. $f(a/w)$: Geometry
2. σ^{∞} : far-field stress
3. a : crack length

Paris law:

$$\boxed{da/dN = C \cdot \Delta K^{m}}$$

$$K = f(a/w) \cdot \sigma \cdot \sqrt{\pi a}$$
$$\Delta K = f(a/w) \cdot \Delta\sigma \cdot \sqrt{\pi a}$$

$$da/dN = C \cdot f(a/w)^{m} \cdot \Delta\sigma^{m} \cdot \pi^{m/2} \cdot a^{m/2}$$

$$\int_{a_i}^{a_c} a^{-m/2}\, da = \int_{0}^{N_f} C \cdot f(a/w)^{m} \cdot \Delta\sigma^{m} \cdot \pi^{m/2} \cdot dN$$

$$C \cdot f(a/w)^{m} \cdot \Delta\sigma^{m} \cdot \pi^{m/2} \cdot N_f$$

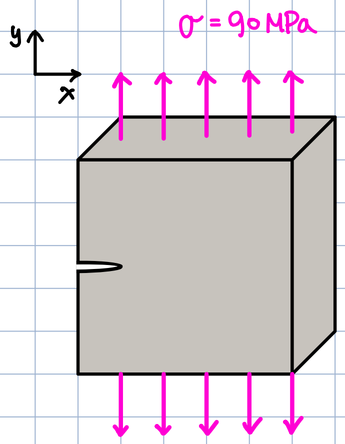
$$\left. a^{-m/2+1} \cdot \frac{1}{-m/2+1} \right]_{a_i}^{a_c}$$

$$= \frac{2}{2-m} \cdot \left(a_c^{\frac{2-m}{2}} - a_i^{\frac{2-m}{2}}\right)$$

$$= \frac{2}{m-2} \cdot \left(\frac{1}{\frac{1}{a_i^{\frac{m-2}{2}}}} - \frac{1}{\frac{1}{a_c^{\frac{m-2}{2}}}} \right)$$

$$\therefore\ N_f = \frac{2}{(m-2)\, C\, f(a/w)^{m} \cdot \Delta\sigma^{m} \cdot \pi^{m/2}} \left(\frac{1}{\frac{1}{a_i^{\frac{m-2}{2}}}} - \frac{1}{\frac{1}{a_c^{\frac{m-2}{2}}}} \right) \quad \text{for } m \neq 2$$

Example >



a fracture toughness : $9.5\,MPa\sqrt{m}$

$C = 6 \times 10^{-11}\,(MPa\sqrt{m})^{m}$

$m = 4$

$K_I = (1.12)\cdot\sigma\sqrt{\pi a}$

initial crack length : 1 mm

10^5 cycles per month

a) What is the critical flaw size ?

b) How many fatigue cycles will this system last?

a) $\Delta K = (1.12)\cdot\Delta\sigma\cdot\sqrt{\pi a}$

$$a_{cr} = \frac{1}{\pi}\cdot\frac{\Delta K^2}{(1.12)^2\,\Delta\sigma^2} = \frac{1}{\pi}\cdot\left\{\frac{9.5\,MPa\sqrt{m}}{(1.12)\cdot(90\,MPa)}\right\}^2$$

$$= 0.028\,m = \underline{28\,mm}$$

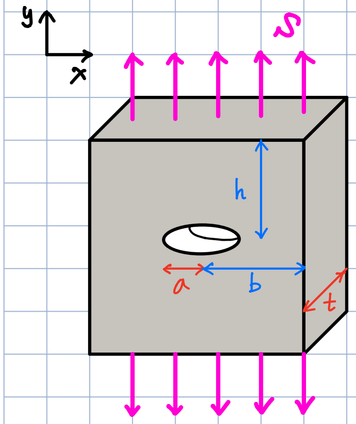
$$\therefore\; N_f = \frac{2}{(m-2)\,C\,f(a/w)^m\cdot\Delta\sigma^m\cdot\pi^{m/2}}\left(\frac{1}{a_i^{\frac{m-2}{2}}} - \frac{1}{a_c^{\frac{m-2}{2}}}\right) \quad \text{for } m \neq 2$$

$$= \frac{2}{(4-2)\cdot(6\times10^{-11})\cdot(1.12)^4\cdot(90)^4\cdot\pi^{4/2}}\cdot\left(\frac{1}{(0.001)^1} - \frac{1}{(0.028)^1}\right)$$

$= 1.57\times10^4$ cycles $\approx$ <u>15.7 months</u> Ans.

Example >

a center-cracked plate : $b = 38\ mm$
$t = 6\ mm$
$a_i = 1\ mm$



cyclic tensile loading : 80–240 kN (freq = 5Hz)

a) What is the critical crack length, $a_c = ?$ ($K_{Ic} = 130\ MPa\sqrt{m}$)

$$\sigma = \frac{S}{2bt} \rightarrow \sigma_{max} = \frac{240\ kN}{(2)(0.038m)\cdot(0.006\ m)} = 526.3\ MPa$$

$$K = f_{(a/w)}\,\sigma\sqrt{\pi a} \rightarrow K_{Ic} = f_{(a/w)}\,\sigma\sqrt{\pi a_c} \rightarrow a_c = \frac{1}{\pi}\cdot\left\{\frac{K_{Ic}}{f_{(a/w)}\cdot\sigma}\right\}^2 \longleftarrow$$ a center cracked plate $f_{(a/w)} = 1$

$$\therefore a_c = \frac{1}{\pi}\cdot\left\{\frac{130\ MPa\sqrt{m}}{(1)\cdot(526.3 MPa)}\right\}^2 = \underline{19.4\ mm}$$ Ans.

b) How many cycles can be applied before failure occurs if life is estimated from its Paris law behaviors ?

$da/dN = C(\Delta K)^m$, $C = 1.095\times10^{-12}\ (da/dN/\Delta K^m)$, $m = 3.24$

$$\Delta K = f_{(a/w)}\,\Delta\sigma\sqrt{\pi a} \longleftarrow \Delta\sigma = \sigma_{max} - \sigma_{min} \longleftarrow \sigma_{max} = 526.3\ MPa$$

$$\sigma_{min} = \frac{80\ kN}{(2)(0.038m)\cdot(0.006\ m)} = 175.4\ MPa$$

$$\therefore \Delta K = (1)\cdot(526.3 - 175.4)\cdot\sqrt{\pi a}$$
$$= 357\ MPa\sqrt{\pi a}$$

Paris law :

$$\boxed{da/dN = C\cdot\Delta K^m} = \left(1.095\times10^{-12}\ \frac{m/cycle}{(MPa\sqrt{m})^{3.24}}\right)\cdot\left(357\ MPa\sqrt{\pi a}\right)^{3.24}$$

$$= \oslash\ a^{1.62}$$

$$\therefore \int_{a_i}^{a_c} a^{-1.62}\,da = \int_0^{N_f} \oslash\ dN$$

$$\int_{1\times10^{-3}}^{19.4\times10^{-3}} a^{-1.62}\,da = \int_0^{N_f} 0.0013\ dN = (0.0013)\cdot N_f$$

$$a^{-0.62}\cdot\frac{1}{-0.62}\Big]_{1\times10^{-3}}^{19.4\times10^{-3}} = (0.0013)\ N_f$$

$$98.26 = (0.0013)\ N_f$$

$\therefore$ $\underline{N_f = 75,562\ cycles}$ Ans.