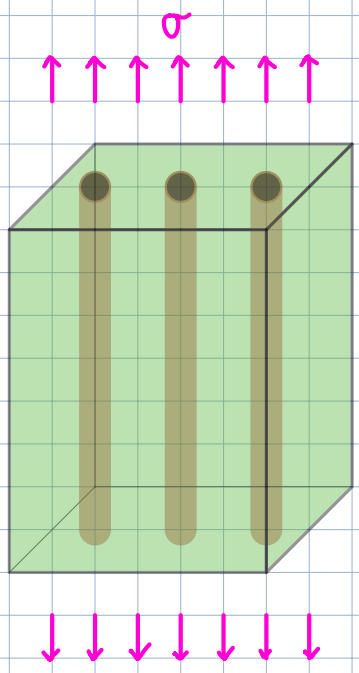
Upper E (Composite)



$\varepsilon = \varepsilon_f = \varepsilon_m$

$A = A_f + A_m$

$F = F_f + F_m$

$F = \sigma A$

$\sigma A = \sigma_f A_f + \sigma_m A_m$

$E\varepsilon A = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m$

$E = E_f \cdot (A_f/A) + E_m \cdot (A_m/A)$

$\therefore$ Upper $E = E_f V_f + E_m V_m$

- Volume fraction

$V_f = A_f/A$

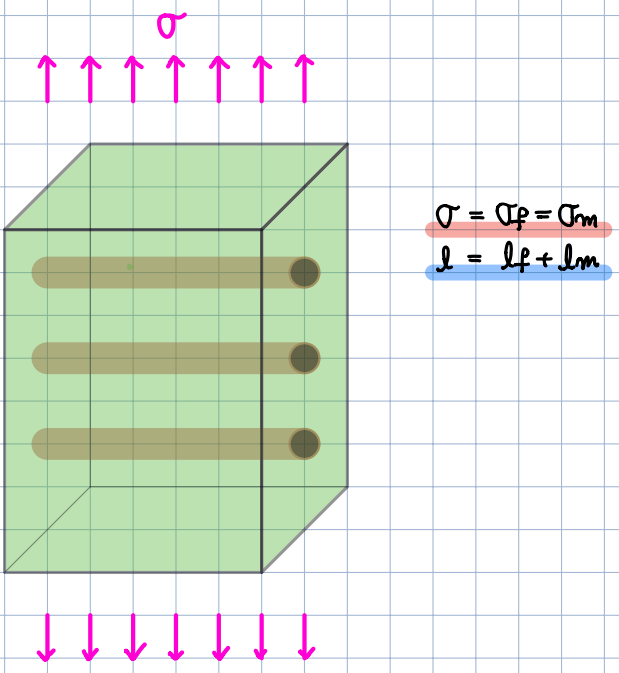
$V_m = A_m/A$

$A_f + A_m = A$

$V_f + V_m = 1$

$V = 1$

Lower E (Composite)



$\sigma = \sigma_f = \sigma_m$

$l = l_f + l_m$

$\varepsilon = \Delta l/l \rightarrow \varepsilon_f = \Delta l_f/l_f$, $\varepsilon_m = \Delta l_m/l_m$

$\Delta l = \Delta l_f + \Delta l_m$

$l\varepsilon = l_f \varepsilon_f + l_m \varepsilon_m$

$\varepsilon = \dfrac{l_f \varepsilon_f + l_m \varepsilon_m}{l}$ $\leftarrow \varepsilon = \sigma/E$

$\sigma/E = \sigma_f/E_f \cdot l_f/l + \sigma_m/E_m \cdot l_m/l$

$1/\text{lower}E = V_f/E_f + V_m/E_m$

$\therefore$ lower $E = \dfrac{E_f \cdot E_m}{E_f V_m + E_m V_f}$

- Volume fraction

$V_f = l_f/l$

$V_m = l_m/l$